Linear-in-T resistivity from van Hove singularity

Keywords: *non-Fermi liquid(NFL), marginal Fermi liquid(MFL), strange metal magnetic heterostructure, optical conductivity, ARPES.*

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Kavli Institute for Theoretical Sciences
University of Chinese Academy of Sciences (UCAS)

Outline

Introduction

- van Hove singularity in cuprates high-T_c superconductors;
- Marginal Fermi liquid and strange metal;
- Theoretical progress on transport: Fermi surface geometry, Yukawa SYK

NFL/MFL from van Hove singularity

- NFL and MFL solutions in different parameter regimes;
- Optical conductivity at zero and finite-T;
- Persistent linear-in-T resistivity.

2D interface of magnetic heterostructure

- VHS + critical triplon at magnetic heterostructure interface;
- ARPES signal directly extracts the NFL feature;
- Spintronics, interfacial DM interactions...

Summary and outlook

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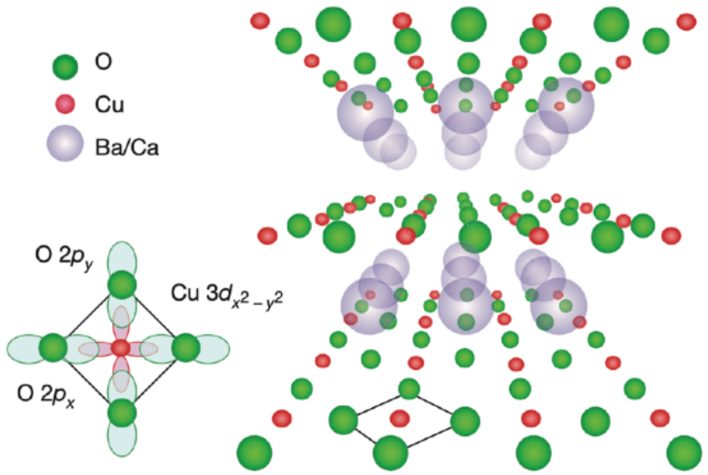
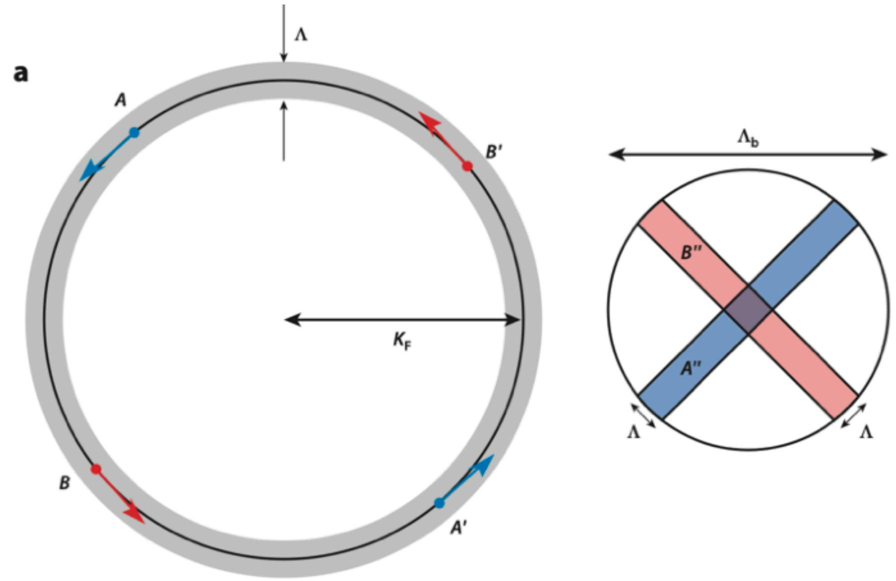
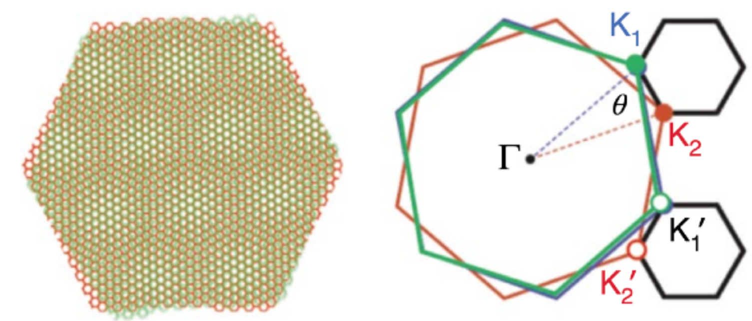
Strong correlation in 2D

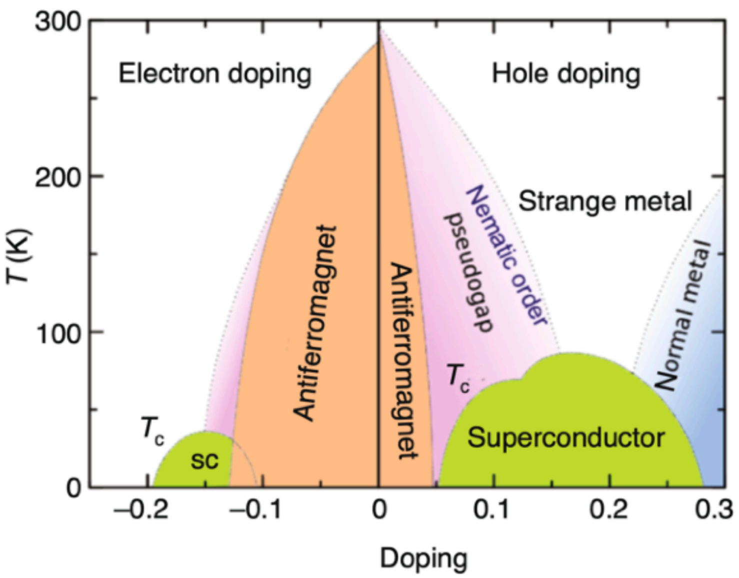


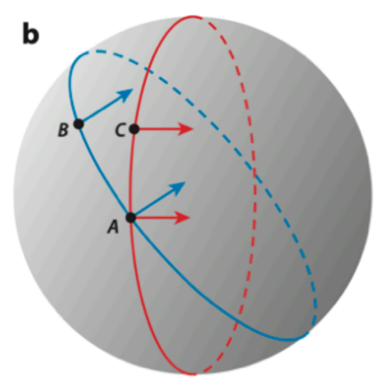
CuO layer in high-T_c SC



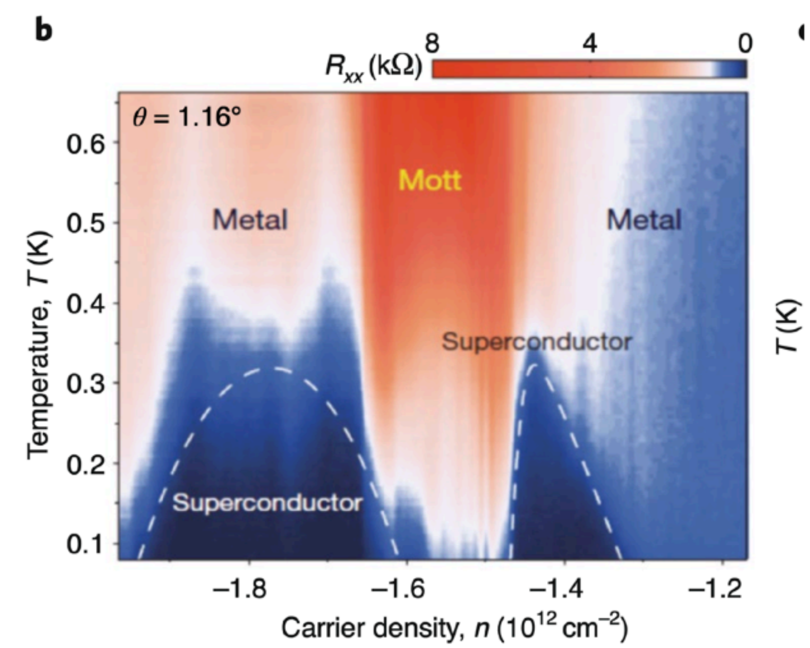


Twisted bilayer graphene





``Patch'' theory for 2D NFLs:
heavy fermions,
itinerant quantum magnets



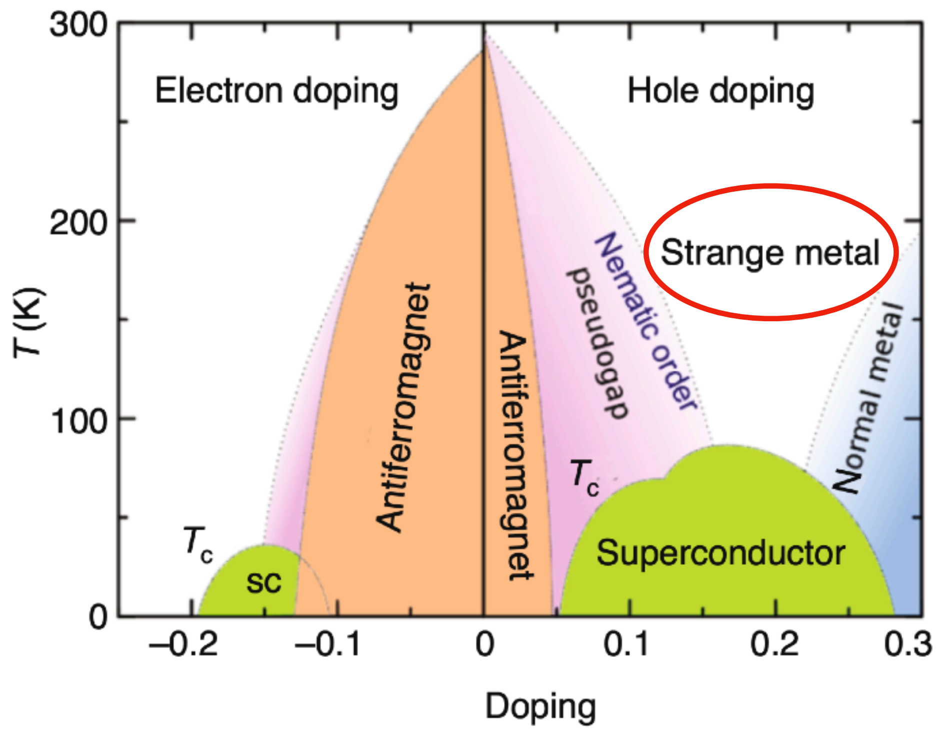
Keimer et al., Nature (2015)

S.-S. Lee,
Annu. Rev. Condens. Matter Phys (2018)

Andrei and Macdonald,
Nat. Mater. (2020)

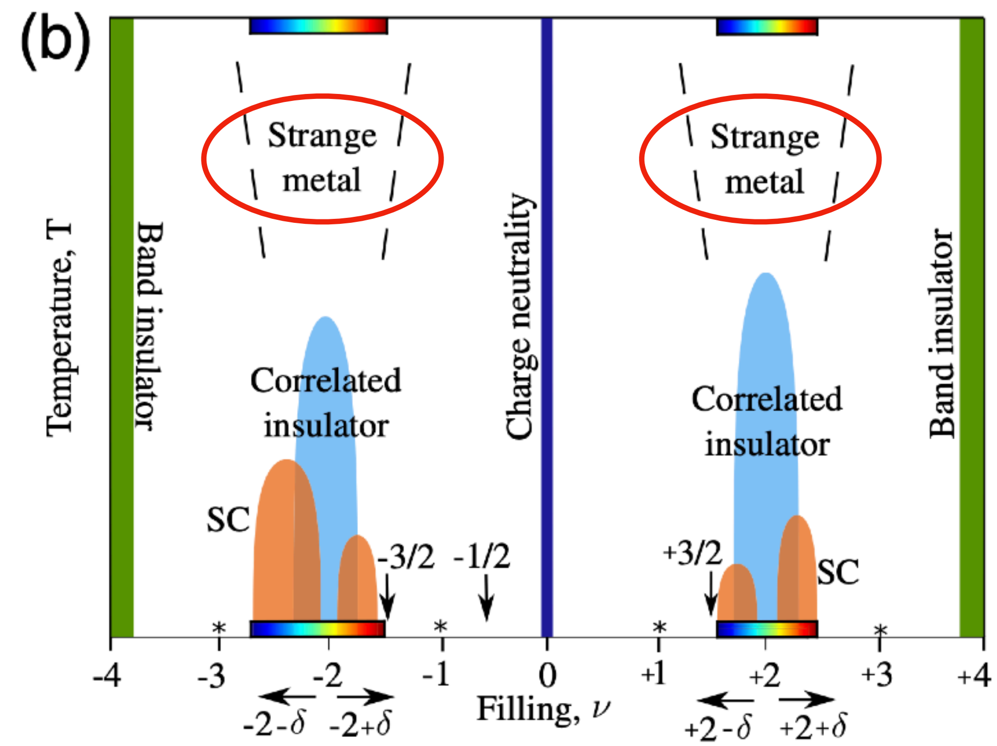
Marginal NFL and strange metal

Y. Cao et al., PRL (2020)



high-T_c cuperate

Keimer et al., Nature (2015)

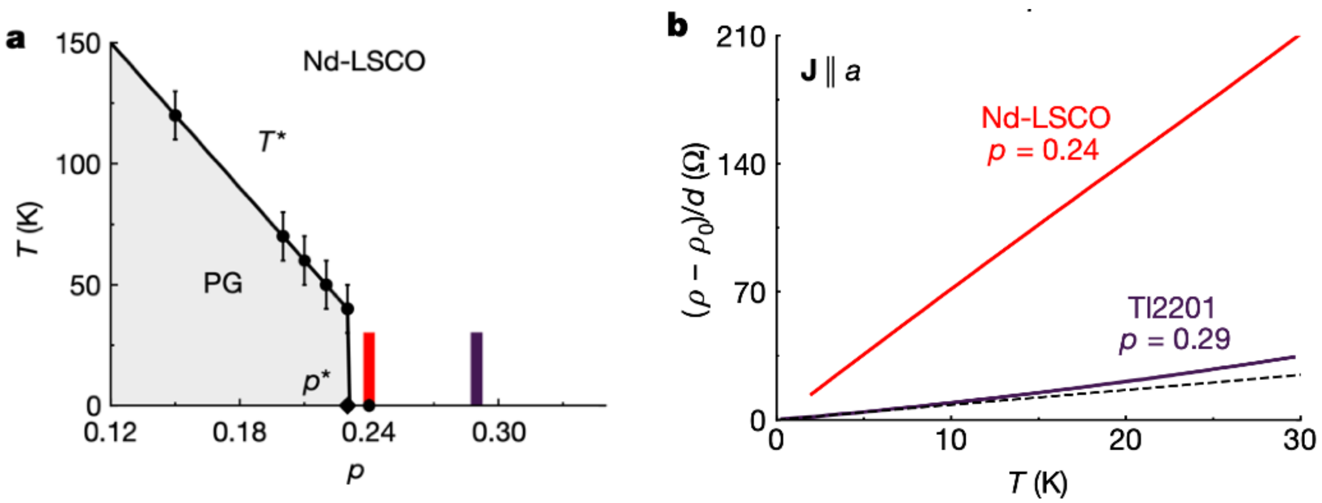


Twisted bilayer graphene

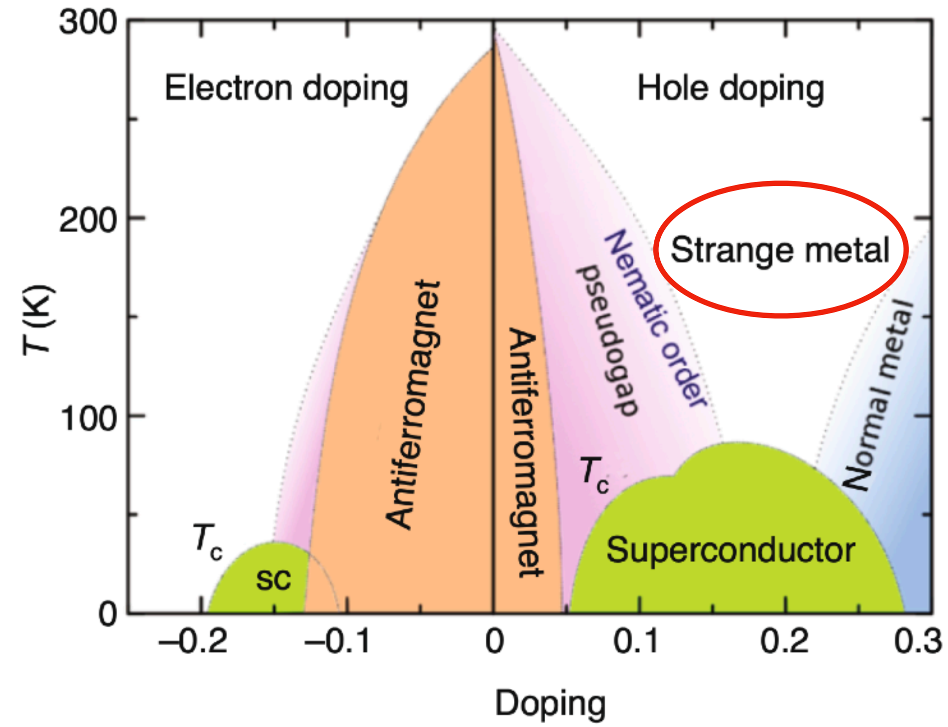
- Strange metal: $\rho(T) = T^{\alpha}, \alpha = 1;$
- Widely found in Cu-based superconductors and twisted bilayer graphene(TBG);
- Phenomenological theory of marginal NFL: $\Sigma(\omega, \boldsymbol{k}_F) \sim \omega \ln(\epsilon_F/\omega) - i\omega.$

C. M. Varma et al., PRB (1989)

Phase diagram: high-T_c cuprates



Taillefer and Ramshaw group, Nature 2021

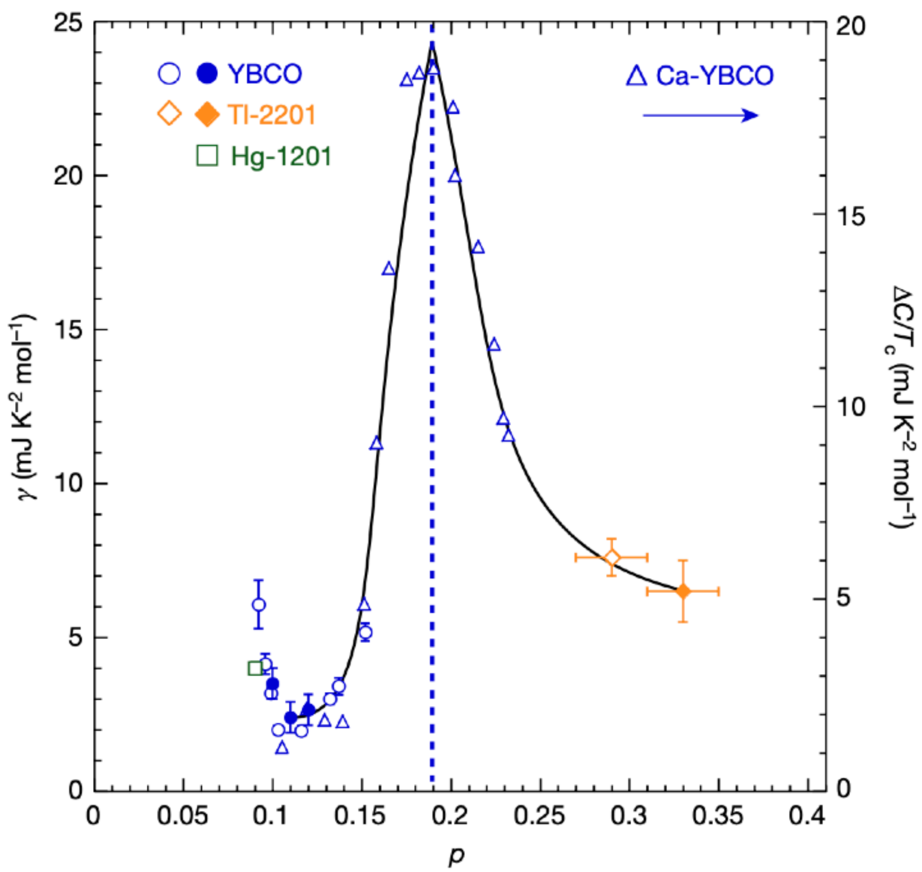


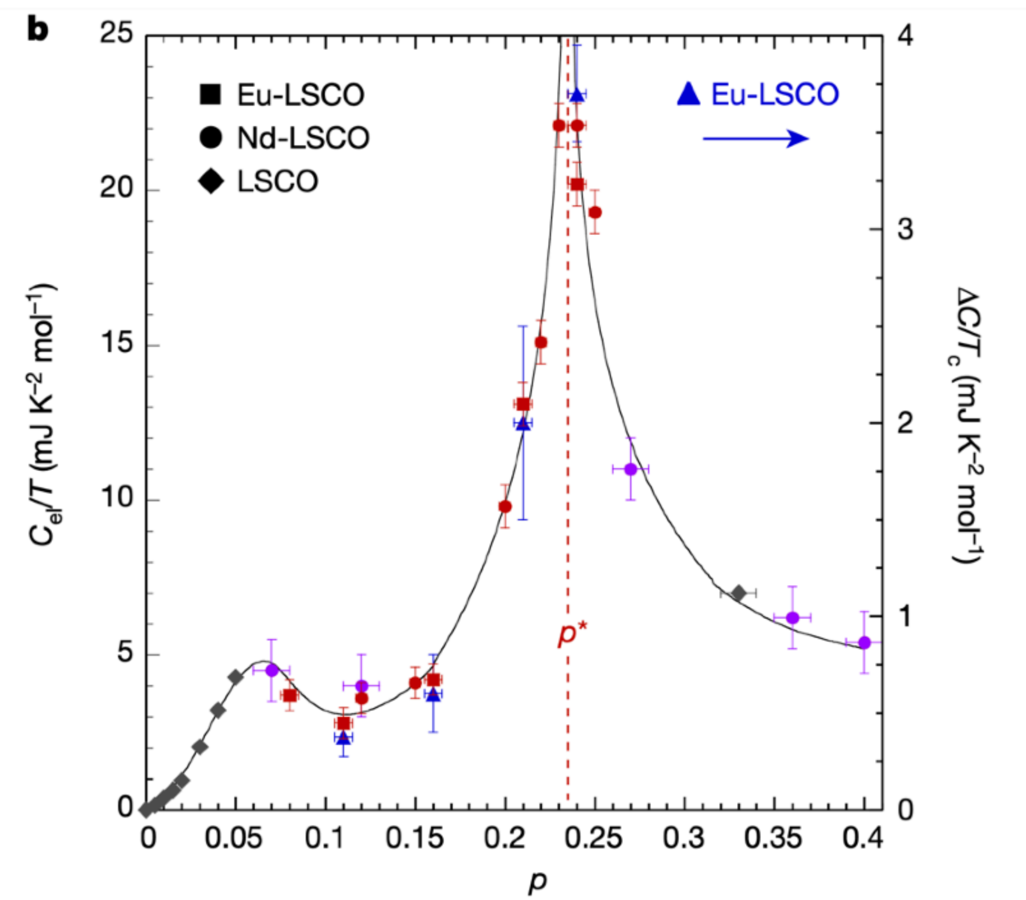
Keimer et al., Nature (2015)

- Pseudogap (PG) phase exists below $T \leq T^\star$: loss of spectral weight;
- PG terminates at a critical hole doping $p = p_c$, which might be a QCP that is masked by the superconducting (SC) dome;
- In the quantum critical fan mediated by the QCP, a universal strange metal behavior is widely observed;
- 3 key ingredients: PG, strange metal, SC, interplay introducing tremendous complexity.
- SC can be killed by applying external fields and a naked QCP is anticipated;
- i)The nature of QCP; ii) its connection to the termination of PG; iii) mechanism for strange metal behaviors are still unclear after decades of investigation.

Fundamental nature of $p = p_c$: *QCP?*

- Thermodynamic signatures, *e.g.* electronic specific heat $C_{\rm el}/T \sim \log(1/T)$, show a log-divergence approaching the critical doping p_c in various cuprates;
- Absence of energy scale ----> Clear evidence for a QCP.





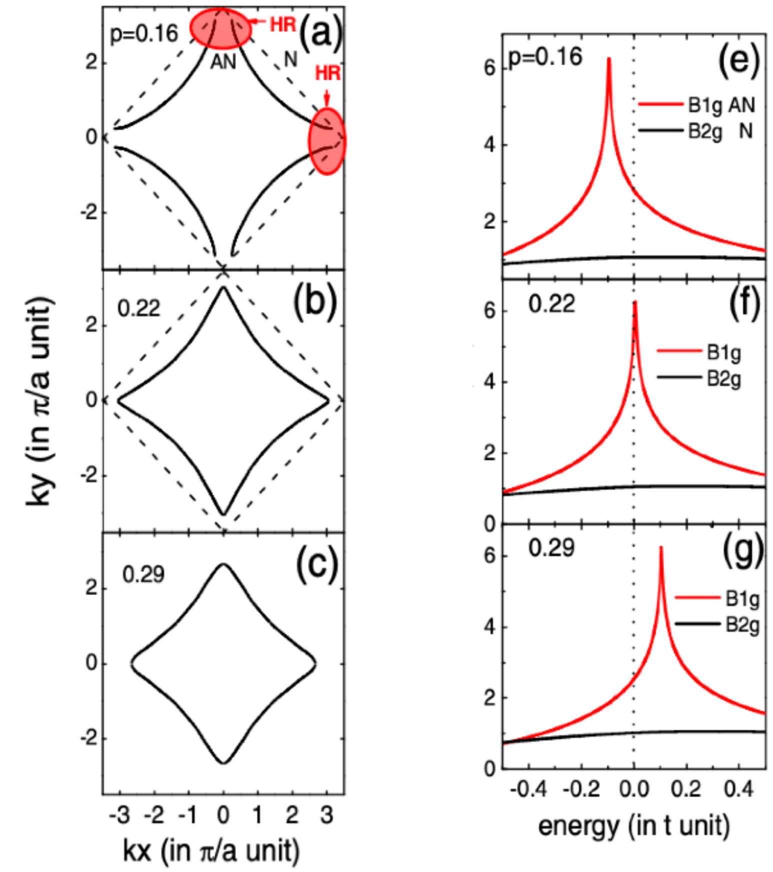
Taillefer and Klein group, Nature 2019

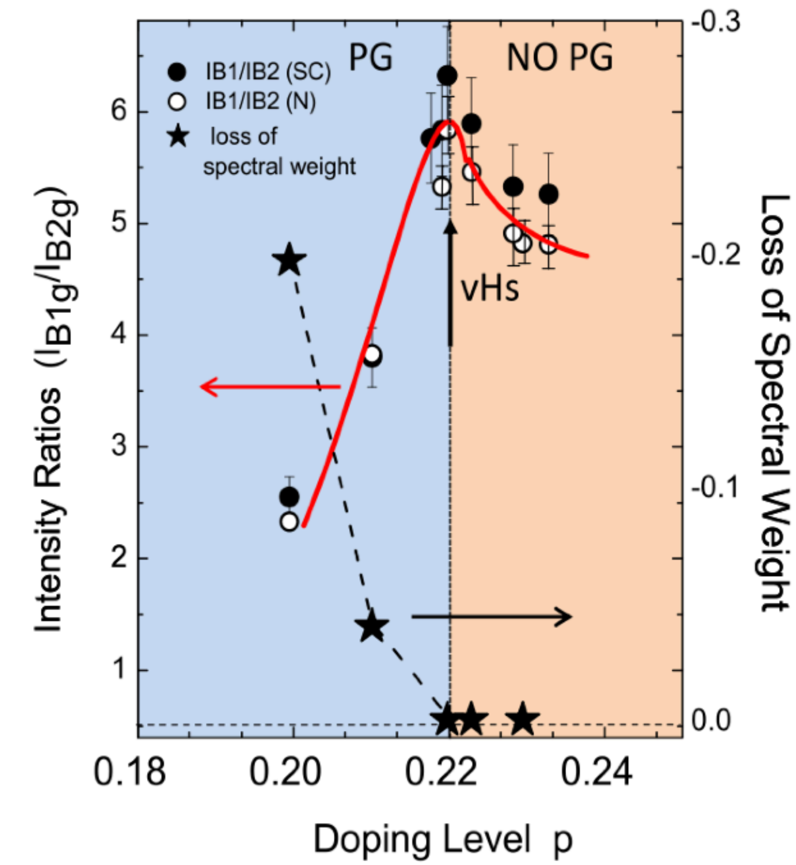
van Hove singularity

incidental or consequential ??

- For many cuprates SC, such as Nd-LSCO [C. E. Matt et al., PRB (2015)] and Ba-2212 [Benhabib et al., PRL (2015)], the PG phase collapses at a Lifshitz transition ----> $p = p_{VHS} \simeq p_c$;

- VHS at the Lifshitz transition -----> divergence in density of state in 2D;



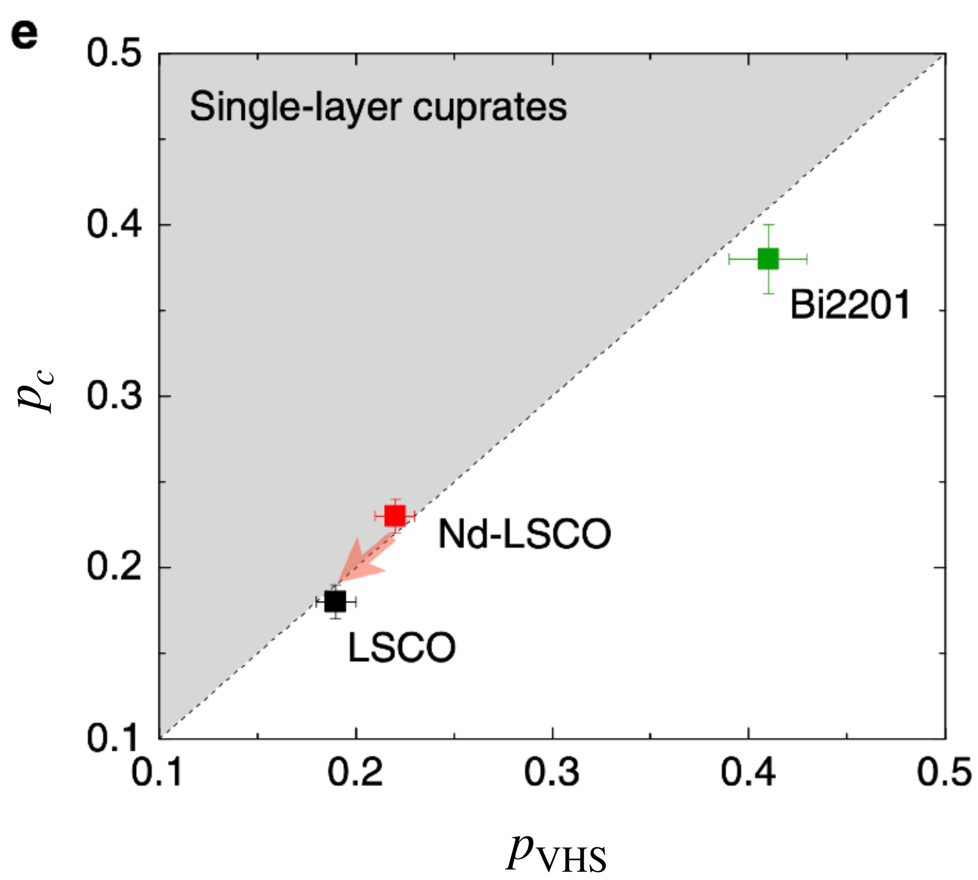


Benhabib et al., PRL (2015)

van Hove singularity

incidental or consequential ??

- Intimate connection between p_c and p_{VHS} in various cuprates: $p_c \leq p_{\mathrm{VHS}}$ compatible with all data so far;
- What determines p_c? coincides with p_{VHS}?
- Quantum critical fluctuation near VHS saddle points play a role in forming strange metal??



Taillefer group, Nat. Comm. (2017)

Controversial role of VHS

PNAS RESEARCH ARTICLE | PHYSICS

Differentiated roles of Lifshitz transition on thermodynamics and superconductivity in $La_{2-x}Sr_xCuO_4$

Yong Zhong[a,b,c,d,1,2], Zhuoyu Chen[a,c,d,1,2,3], Su-Di Chen[a,c,d], Ke-Jun Xu[a,c,d], Makoto Hashimoto[e], Yu He[f], Shin-ichi Uchida[g], Donghui Lu[e], Sung-Kwan Mo[b], and Zhi-Xun Shen[a,c,d,h,2]

Contributed by Zhi-Xun Shen; received March 17, 2022; accepted June 7, 2022; reviewed by Peter Armitage and Antony Carrington

1. VHS at the band structure level is already enough for the explanation of thermodynamic divergence and SC;

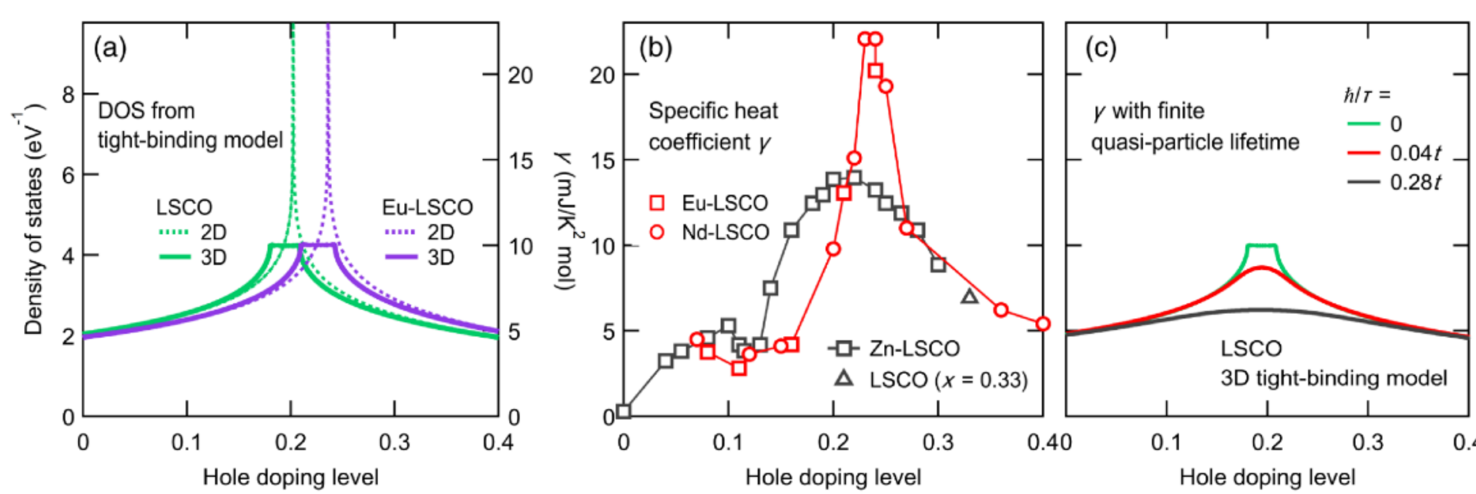
Zhi-Xun Shen group, PNAS (2022) ......

No need to consider the QCP and associated quantum fluctuation.

2. LSCO has a k_z-dispersion. 3D nature suppresses the VHS making it incapable to explain the experiments

The quantum fluctuation near the QCP is important.

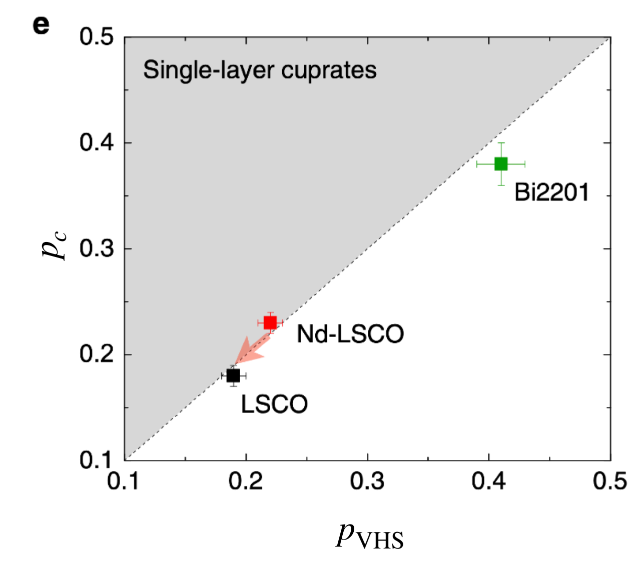
M. Horio et al., PRL (2018)



Important role of VHS

2. LSCO has a k_z-dispersion. 3D nature suppresses the VHS making it incapable to explain the experiments

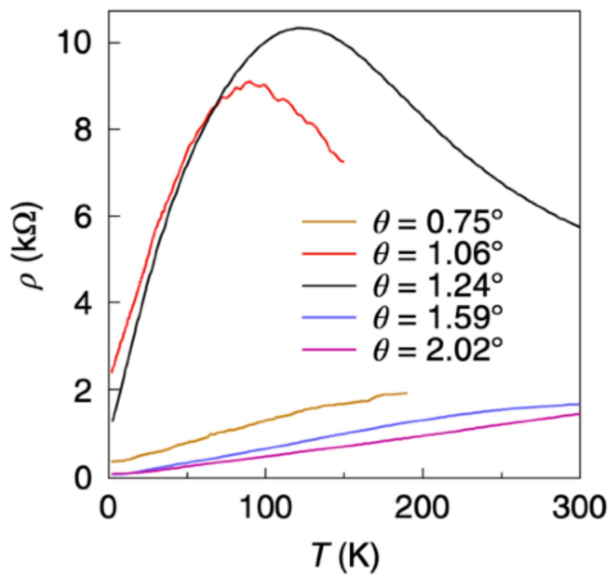
The quantum fluctuation near the QCP is important.

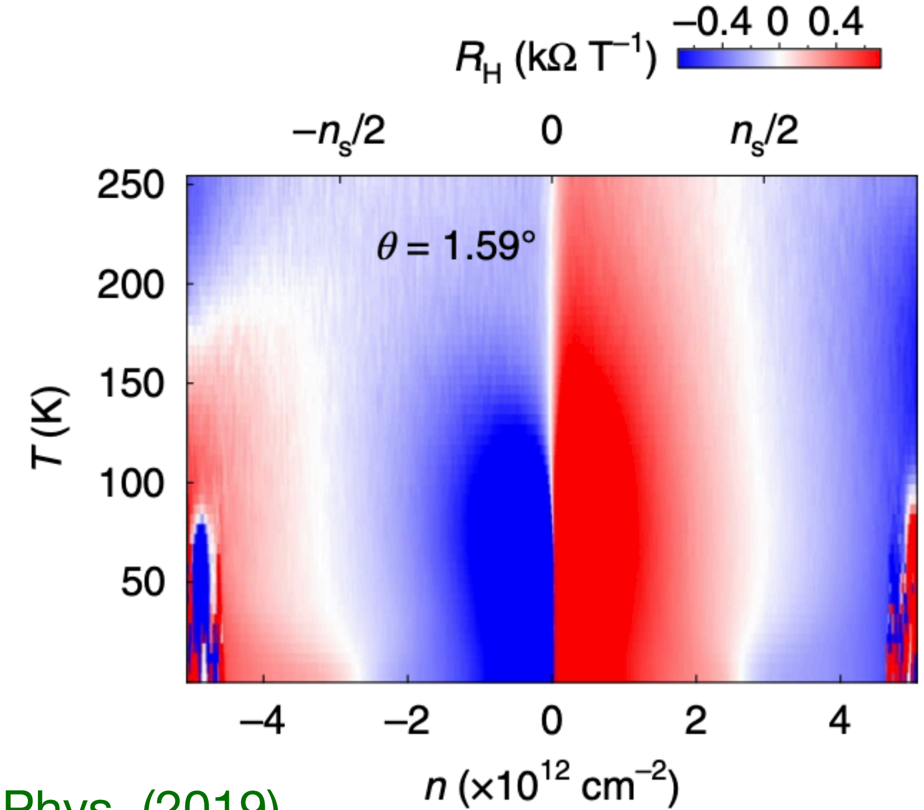


Question:

- What are the quantum critical phenomenons associated with VHS?
- Will there be any strange metal behaviors near the Lifshitz transition when the Fermi surface is coupled with critical bosons?

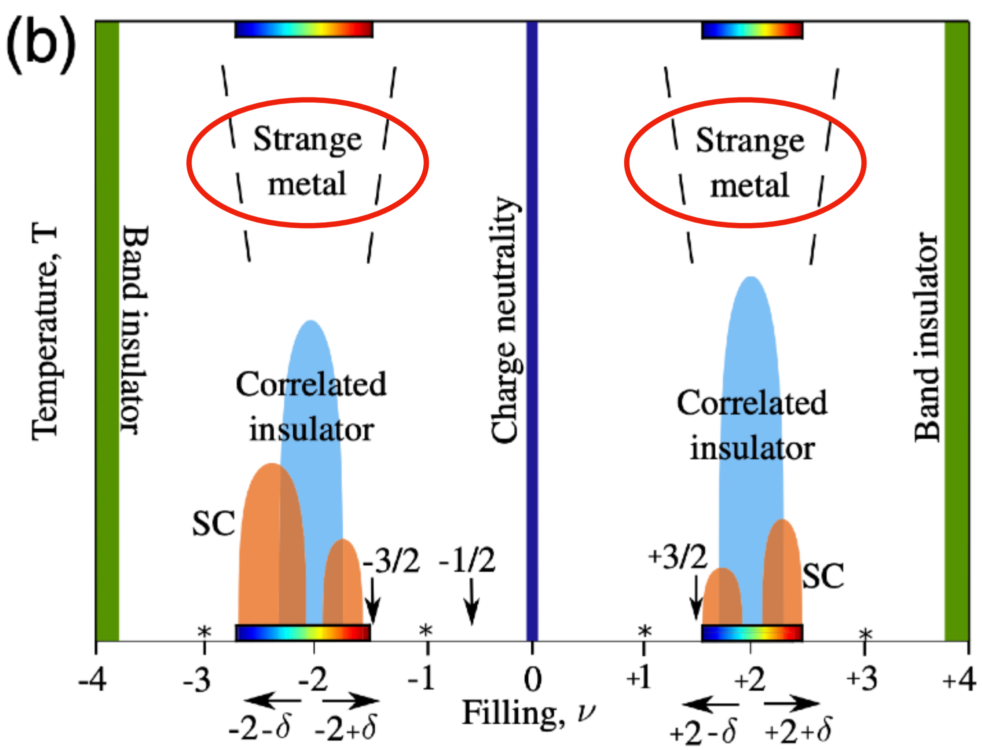
Strange metal in TBG

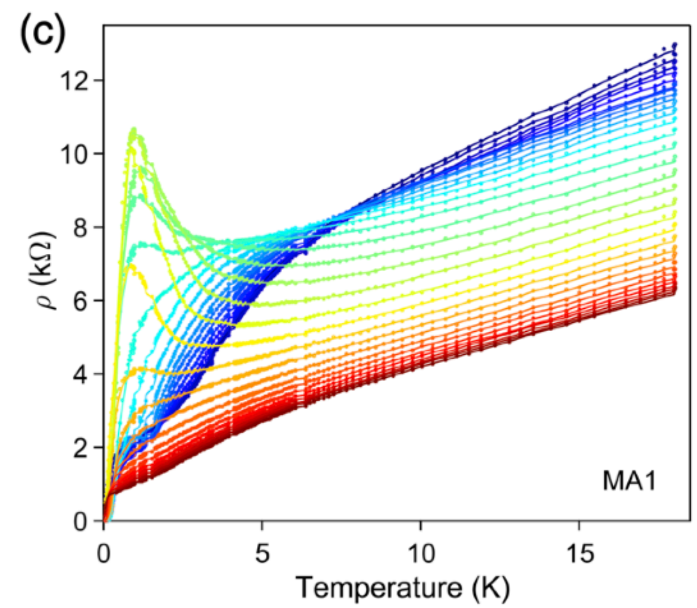


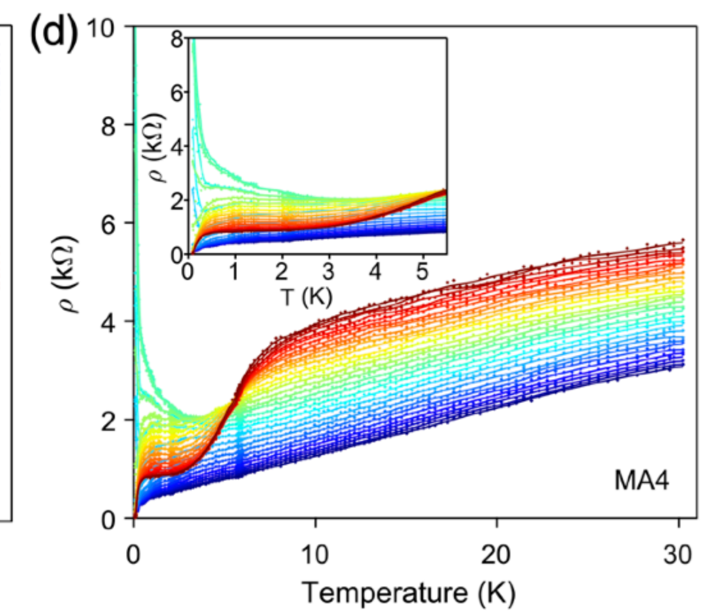


Andrea Young Group, Nat. Phys. (2019)

- Hall resistivity changes sign at $n = \pm n_s/2$
  a feature associated with the Liftshitz transition with VHS.







Y. Cao et al., PRL (2020)

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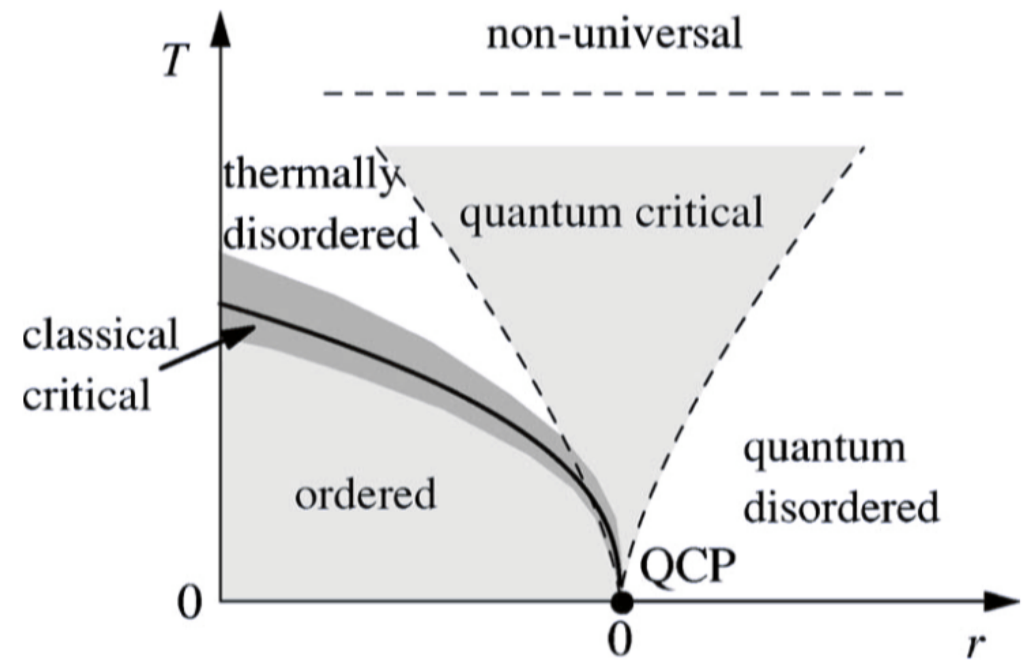
NFL induced at QCP



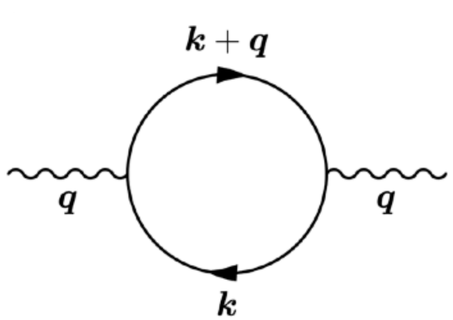
- Fermion-boson coupled model:

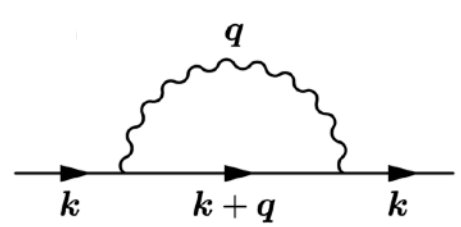
$$H_{\rm NFL} = \int d^2\boldsymbol{r}\Big[c^\dagger_\alpha(\boldsymbol{r})\epsilon(-i\partial_{\boldsymbol{r}})c_\alpha(\boldsymbol{r})$$
$$-\lambda O(\boldsymbol{r})\phi(\boldsymbol{r}) + \frac{1}{2}\left(\partial_r\phi\right)^2 + \frac{r}{2}\phi^2 + \ldots\Big].$$

$O(\mathbf{r})$ represents a fermionic bilinear;
ϕ is the order parameter in the Ginzburg-Landau expansion.

- Two gapless degrees of freedom are coupled: extreme strong interaction !
- Phenomenological theory: Hertz-Millis method;
- Fermion large-N: may not commute with $\omega \to 0$

A. J. Hertz (1976)
A. Millis (1993)





Polchinski (1991)
P. A. Lee and Nagaosa (1992)
Nayak and Wilczek (1994);

Controlled perturbation theory in 2D

- Large-N expansion: infinite many diagrams [Sung-Sik Lee, PRB (2009)];
- Double expansion in dynamic exponent z and N: non-local interactions for fermions [D. F. Mross et al., PRL (2010)];
- Hertz-Millis theory: biased and dangerous phenomenological theory favoring fermions; gapless fermions are integrated out completely! not a RG;
- Momentum shell RG is developed by treating fermions and bosons on equal footing [Fitzpatrick et al., PRB (2013-14)]

- Difficulty in developing a controlled microscopic theory ----->
  new theoretical method from quantum gravity: Sachdev-Ye-Kitaev (SYK) model;
- The Yukawa-SYK model has been demonstrated as a controlled expansion in large-N;
- Universal theory for the strange metal behaviors;
- Microscopic origin is NOT clear.

Esterlis and Schmalian, PRB (2019)
Yuxuan Wang, PRB (2020)
Esterlis, Guo, Patel and Sachdev, PRB (2021)
Guo, Patel, Esterlis and Sachdev, PRB (2022)
Patel, Guo, Esterlis and Sachdev, Science (2023)

Non-perturbative methods

- Quantum anomaly and anomaly assisted perturbation [Z. D. Shi et al., SciPost (2022,2023)]
- Bosonization of the Fermi surface [D. T. Son, PRR (2022)]

MFL from randomness

Yukawa SYK model

Esterlis and Schmalian, PRB (2019)
Esterlis, Guo, Patel and Sachdev, PRB (2021)
Guo, Patel, Esterlis and Sachdev, PRB (2022)
Patel, Guo, Esterlis and Sachdev, Science (2023)

$$\mathcal{S}_g = \int d\tau \sum_{\mathbf{k}} \sum_{i=1}^{N} \psi^\dagger_{i\mathbf{k}}(\tau)[\partial_\tau + \varepsilon(\mathbf{k})]\psi_{i\mathbf{k}}(\tau) + \frac{1}{2}\int d\tau \sum_{\mathbf{q}} \sum_{i=1}^{N} \phi_{i\mathbf{q}}(\tau)(-\partial_\tau^2 + K\mathbf{q}^2 + m_b^2)\phi_{i,-\mathbf{q}}(\tau) + \frac{g_{ijl}}{N}\int d\tau d^2 r \sum_{i,j,l=1}^{N} \psi^\dagger_i(\mathbf{r},\tau)\psi_j(\mathbf{r},\tau)\phi_l(\mathbf{r},\tau)$$

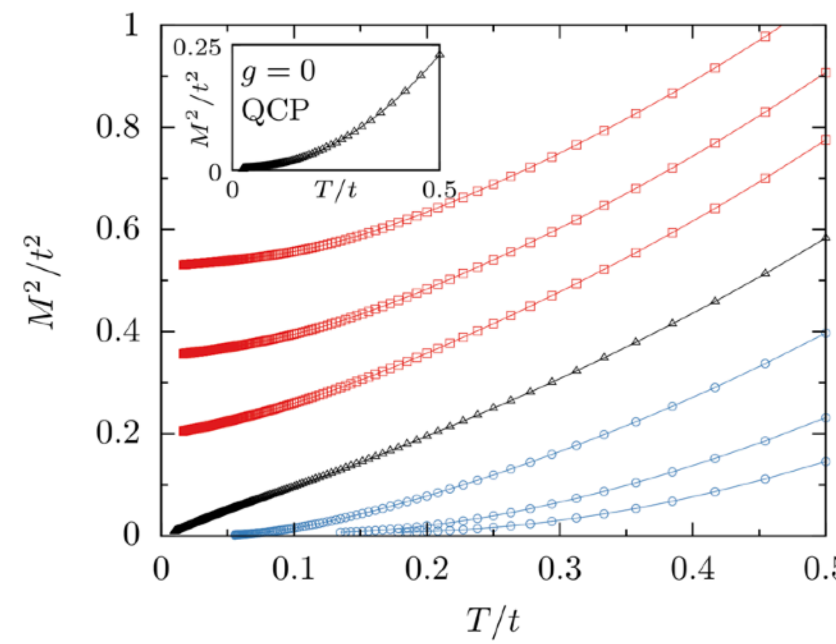
- Fermi surface coupled to critical boson ----> NFL by destroying the quasiparticle;
- Strong coupled fermion-boson system in 2D renders the usual perturbation scheme (such as Large-N expansion) not applicable; Sung-Sik Lee, PRB (2009)
- Introducing the flavor randomness in g_{ijk} Yukawa interaction -----> Self-consistent NFL solution but *NOT* a strange metal;
- Fermi surface coupled to critical boson with potential disorder $v_{ij}(\mathbf{r})$----> MFL but *NOT* a strange metal;
- Fermi surface coupled to critical boson with spatially random Yukawa interaction $g'_{ijk}(\mathbf{r})$ ----> MFL + a strange metal.

Fermi surface coupled with critical boson

Yukawa SYK with spatially uniform interaction g

Esterlis, Guo, Patel and Sachdev, PRB (2021)

$$
\begin{aligned}
S_g = & \int d\tau \sum_{\mathbf{k}} \sum_{i=1}^{N} \psi^\dagger_{i\mathbf{k}}(\tau)[\partial_\tau + \varepsilon(\mathbf{k})]\psi_{i\mathbf{k}}(\tau) + \\
& \frac{1}{2}\int d\tau \sum_{\mathbf{q}} \sum_{i=1}^{N} \phi_{i\mathbf{q}}(\tau)(-\partial_\tau^2 + K\mathbf{q}^2 + m_b^2)\phi_{i,-\mathbf{q}}(\tau) + \\
& \frac{g_{ijl}}{N}\int d\tau d^2 r \sum_{i,j,l=1}^{N} \psi_i^\dagger(\mathbf{r},\tau)\psi_j(\mathbf{r},\tau)\phi_l(\mathbf{r},\tau)
\end{aligned}
$$



- Large-N saddle point solution from the disorder averaged action:

$$
\Pi(i\omega,\mathbf{q}) = -c_b\frac{|\omega|}{|\mathbf{q}|}, \Sigma(i\omega,\mathbf{k}) = -ic_f\,\mathrm{sgn}(\omega)|\omega|^{2/3}
$$

- Boson thermal mass: $m_b^2(T) \equiv m_b^2 - \Pi(0,0)$;

  Quantum critical region: $m_b^2(T) \sim T$, consistent with Hartnoll et al., PRB (2014)

  Fermi liquid phase: $m_b^2(T) \sim T^2$

Guo, Patel, Esterlis and Sachdev, PRB (2022)

- Optical conductivity: Only a Drude peak due to no current relaxation;

  Previously proposed $\sigma(\omega) \sim C\omega^{-2/3}$ [Y. B. Kim, A. Furusaki, X.-G. Wen, and P. A. Lee , PRB (1994)] is shown to be vanishing $C = 0$ due to various cancellations for a generic convex Fermi surface.

Fermi surface coupled with critical boson

Yukawa SYK with spatially random interaction g'

Patel, Guo, Esterlis and Sachdev, Science (2023)

- Spatially uniform interaction g_{ijk} + fermion potential disorder $v_{ij}(\mathbf{r})$ leads to MFL self-energy $\Sigma_g(\mathbf{k}_F, \omega) \sim i\dfrac{g^2}{v^2}\omega \ln|\omega| + \ldots$ but NOT linear-in-T resistivity;
- One relies on the spatially random interaction $g'_{ijk}(\mathbf{r})$ to generate strange metal transport.

- Large-N saddle point solution from the disorder averaged action:

$$\Pi(\mathbf{q}, \omega) = -\frac{g^2}{2\pi v^2}|\omega| - \frac{\pi D_F^2}{2} g'^2 |\omega| \equiv c_d |\omega|,$$

$$\Sigma(\mathbf{k}_F, \omega) = \Sigma_g(\mathbf{k}_F, \omega) - i\frac{D_F g'^2 \omega}{4\pi} \ln(c_d|\omega|)$$

Fermi surface coupled with critical boson

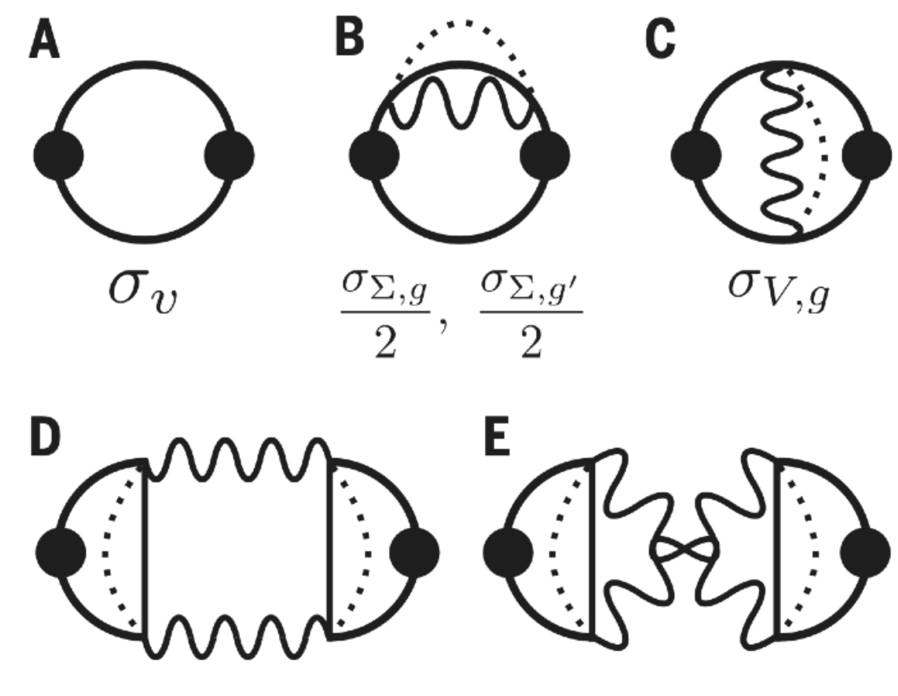
Yukawa SYK with spatially random interaction g'

Patel, Guo, Esterlis and Sachdev, Science (2023)

- Optical conductivity: perturbative expansion in $o(g^2, g'^2)$ in large-N limit

$$\mathrm{Re}[\sigma(\omega \gg T)] = \sigma_{\Sigma,g} + \sigma_{V,g} + \sigma_{\Sigma,g'} \sim -g'^2|\omega|$$

- Renormalized boson propagator ($z = 2$): $D^{-1}(\mathbf{q}, \omega) = q^2 + c_d|\omega| + m_b^2(T)$ with the boson thermal mass $m_b^2(T) \simeq T \sim |\omega| \sim q^2$



$\sigma_{AL_1,g} + \sigma_{AL_2,g} = 0$

- Strange metal: $\rho(T \gg |\omega|) \sim T$ is inferred from scaling rule $|\omega|/T$.

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Non-Fermi liquids at VHS

Y.-H. Xing, W. Liu, ***XTZ***[†], arXiv: 2407.02270.

Yi-Hui Xing (邢益辉)
PhD student at
Institute of Physics, CAS

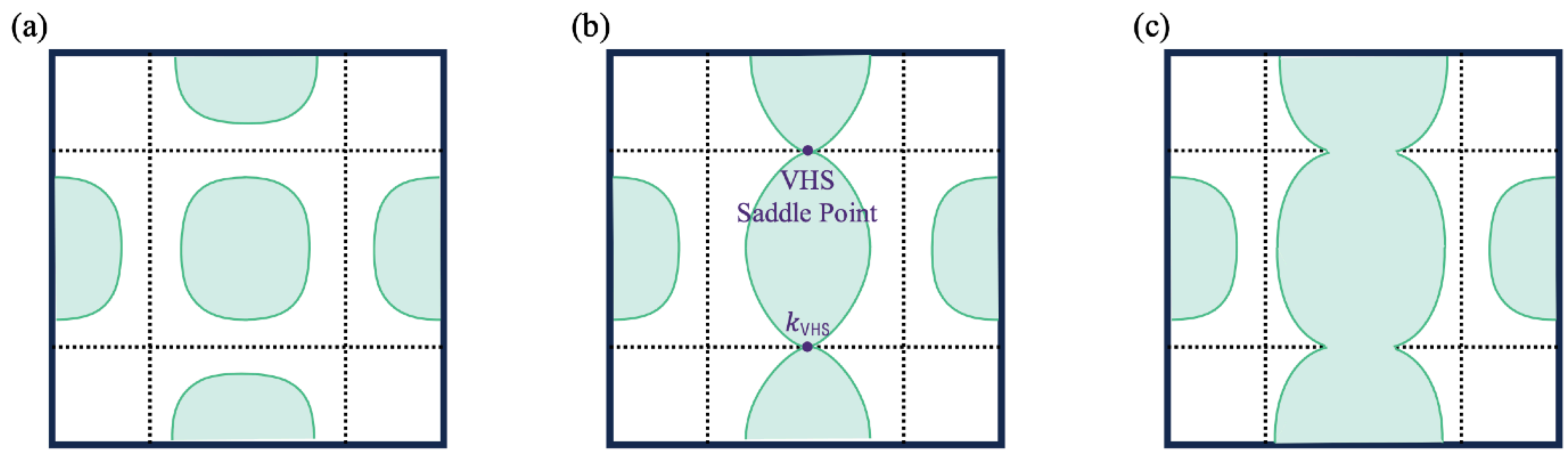


Fig. 2 The geometric evolution of 2D Fermi surface under external fields. Solid green lines in **a** and **c** are the generic convex and concave Fermi surface, respectively. **b** stands for the topological Lifshitz transition point with VHS saddle points (purple dots at $k_{\rm V}$) which represents the convex-concave geometric transition.

- Low-energy effective theory: VHS + critical bosons

$$\mathcal{L} =\psi_m^\dagger(\partial_\tau - \partial_x^2 + a\partial_y^2)\psi_m + \phi_l(\Delta - \partial_\tau^2 - \boldsymbol{\nabla}^2)\phi_l + \frac{1}{\sqrt{NN'}}\left[J_{mnl} + J'_{mnl}(\boldsymbol{x})\right]\psi_m^\dagger\psi_n\phi_l + h.c.,$$

where $m, n = 1,2,...N$ and $l = 1,2,...N'$ are fermionic and bosonic flavor indexes, respectively.

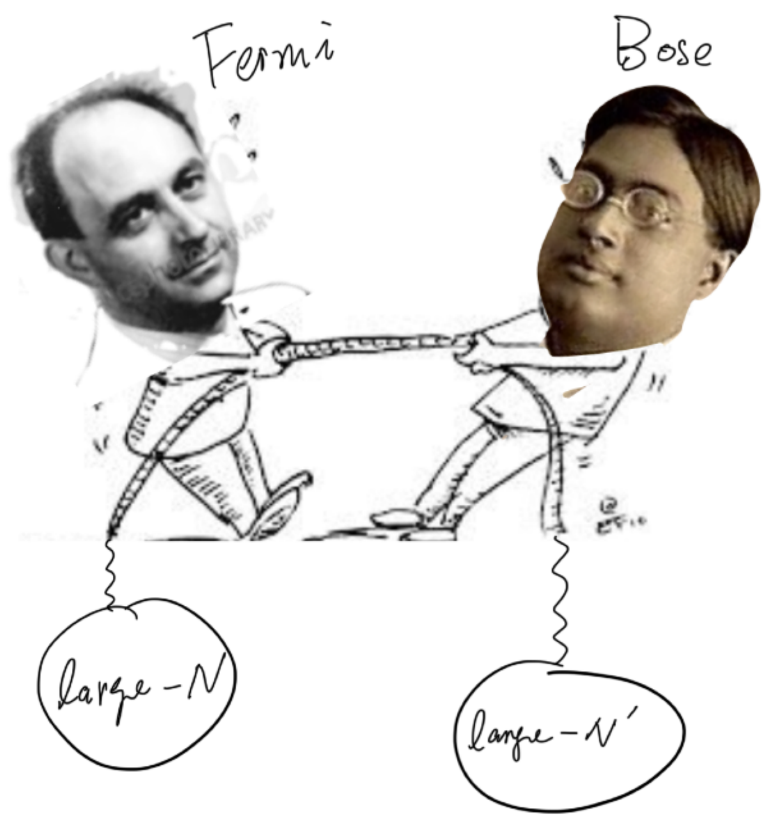
NFL/MFL solution

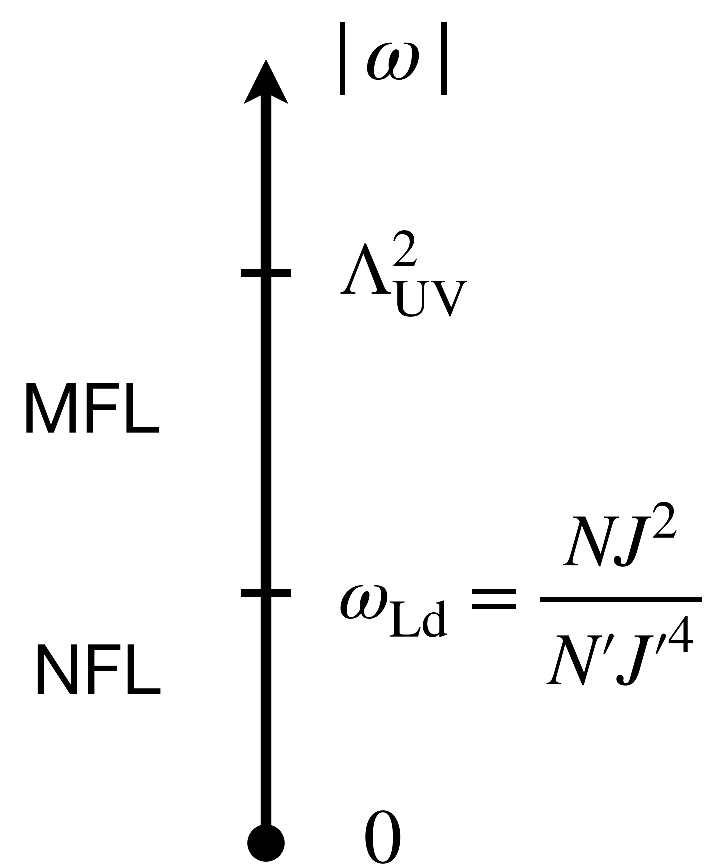
- At finite energies $\omega_{\mathrm{Ld}} < |\omega| \ll \Lambda_{\mathrm{UV}}^2$, we obtain the MFL solution

$$\Pi(i\Omega_m, \boldsymbol{q}) - \Pi(0) \simeq -\frac{|J'|^2\Lambda^2}{2\pi^3}\frac{N}{N'}|\Omega_m|,$$ overdamped boson

$$\Sigma(i\omega_n, \boldsymbol{k}) - \Sigma(0) \simeq i\frac{|J'|^2\Lambda}{8\pi^3}\omega_n \ln|\omega_n|,$$ MFL self-energy

where Π, Σ are self-energy corrections for bosons and fermions





NFL/MFL solution

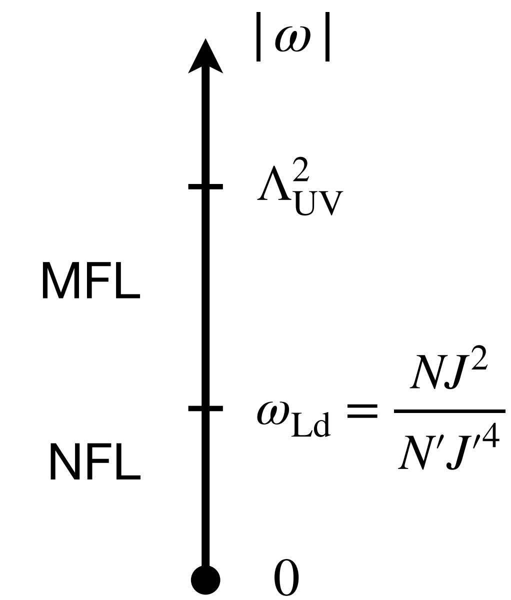
- NFL in low-energy limit $|\omega| < \omega_{\rm Ld}$

$$\Pi(i\Omega_m, \boldsymbol{q}) - \Pi(0) \simeq -\frac{|J|^2}{4\pi}\frac{N}{N'}\frac{|\Omega_m|}{|q_x^2 - aq_y^2|},$$

Landau damping

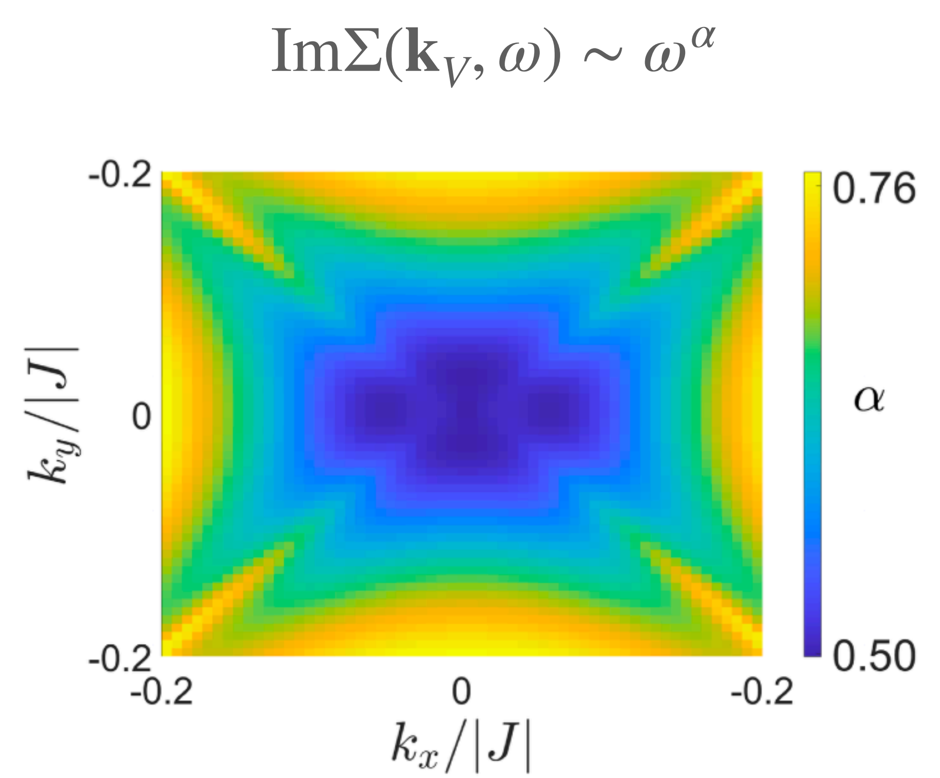
$$\Sigma(i\omega_n, \boldsymbol{k}) - \Sigma(0) \simeq -i\frac{|J|}{4\sqrt{\pi}}\sqrt{\frac{N'}{N}}\mathrm{sgn}(\omega_n)|\omega_n|^{1/2}.$$

NFL at VHS



- NFL self-energy at VHS dominates over the rest of FS points: $\mathrm{Im}\Sigma(\mathbf{k}_F, \omega) \sim \omega^{2/z}$ for $z < 4$;

- The fermion self-energy is enhanced by a factor $\sqrt{N'/N}$ (compared with the MFL case).



Outline

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NFL/MFL from van Hove singularity

- NFL and MFL solutions in different parameter regimes;
- Optical conductivity at zero and finite-T;
- Persistent linear-in-T resistivity.

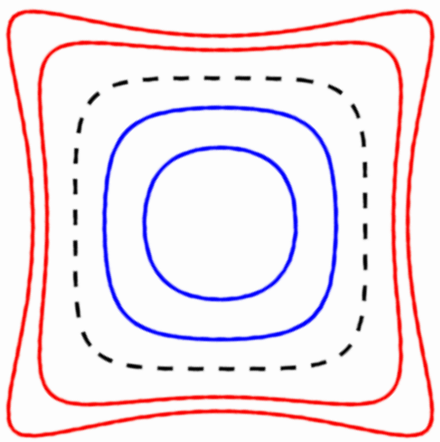
2D interface of magnetic heterostructure

- VHS + critical triplon at magnetic heterostructure interface;
- ARPES signal directly extracts the NFL feature;
- Spintronics, interfacial DM interactions...

Summary and outlook

Optical conductivity

- Optical conductivity $\sigma(\omega, T = 0)$ probes the electronic scattering rate.
- At low frequency (i.e. dc-limit), the optical conductivity comes mainly from Umklapp process, phonon, impurity scatterings; while at high frequency, the transport scattering is dominated by the normal electron interaction.
- Two principals for $\sigma(\omega) \neq 0$: i) momentum relaxation mechanism; OR ii) Galilean invariance breaking: Maslov and Chubukov, Rep. Prog. Phys. (2017)
- Parabolic dispersion is isotropic and preserves Galilean invariance ----> $\sigma(\omega) = 0$; non-parabolic yet isotropic dispersion, *e.g.* Dirac metals, leads to a suppressed but finite $\sigma(\omega) \neq 0$;
- Fermi surface geometry: 2D convex Fermi surface ---> $\sigma(\omega) = 0$ from perturbative [Guo, Patel, Esterlis and Sachdev, PRB (2022)] and non-perturbative [Shi, Goldman, Else and Senthil, SciPost (2022)] methods;
- Concave Fermi surface in 2D ----> $\sigma(\omega) \neq 0$:
  Songci Li ... and Maslov, PRB (2023); Gindikin and Chubukov, PRB (2024)
- Convex-to-concave transition is explored:
  Songci Li ... and Maslov, PRB (2023); Hao Song ... and Sung-Sik Lee, arXiv 2023.



Optical conductivity of VHS at $T = 0$

- Kubo formula for the optical conductivity: $\mathrm{Re}[\sigma(\Omega, T)] = \dfrac{\mathrm{Im}[\Xi^R(\Omega, k=0)]}{\Omega}$

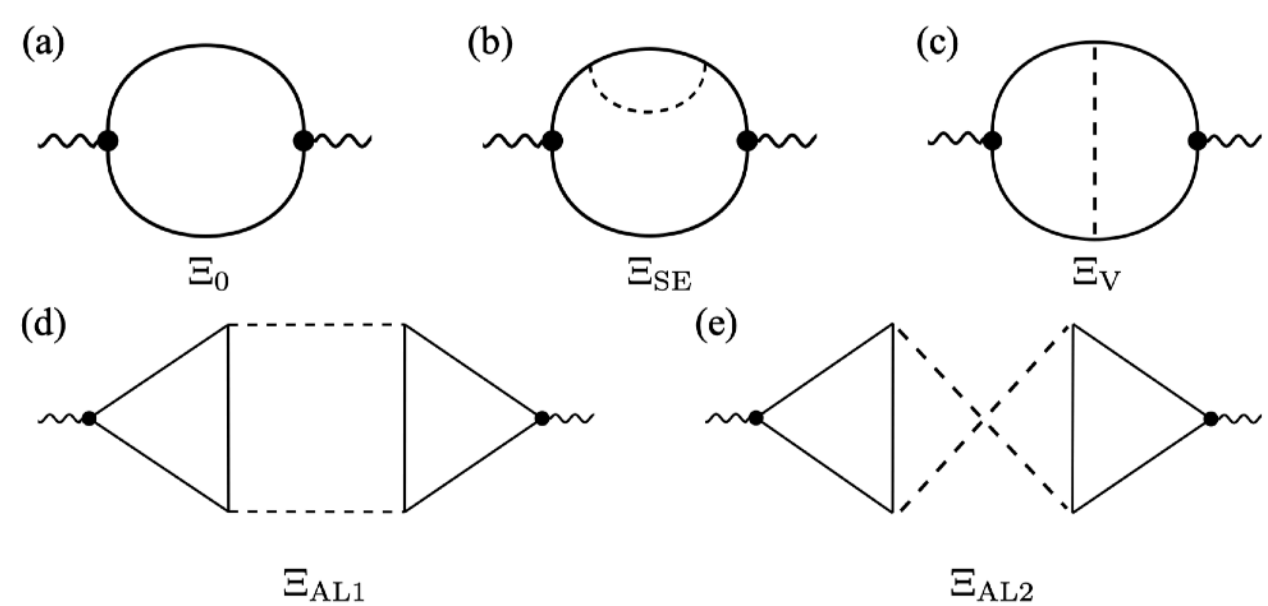
  where Ξ^R is the retarded current-current correlation function.

- Perturbative calculation in $o(J^2, J'^2)$ in large-N limit

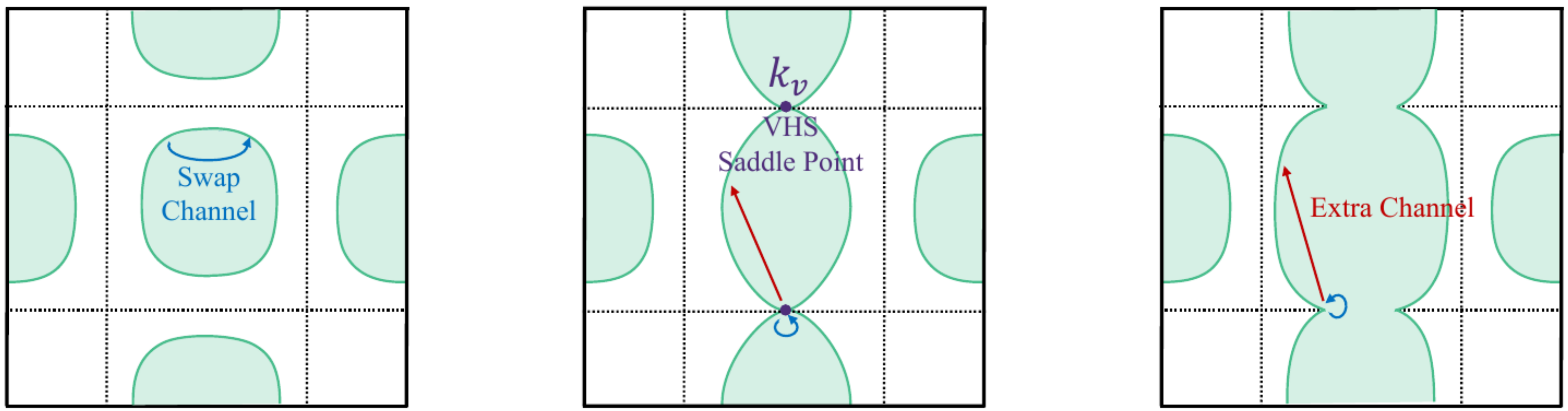
  where J, J' are the spatially uniform, random interaction, respectively;

- The overall conductivity can be divided into two parts:

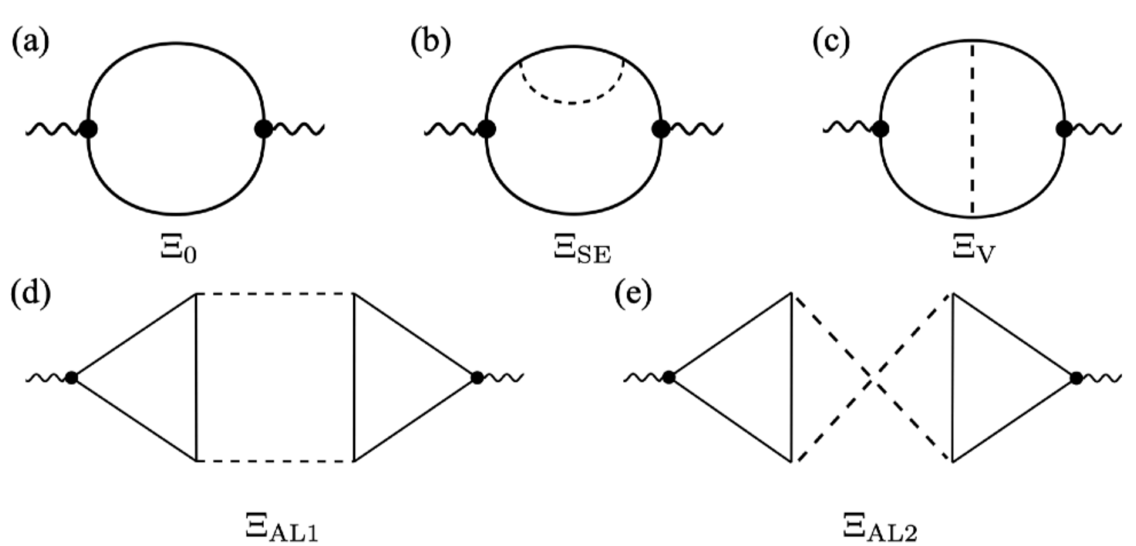
  $\mathrm{Re}[\sigma(\Omega, T)] = \mathrm{Re}[\sigma^J]$+$\mathrm{Re}[\sigma^{J'}]$.

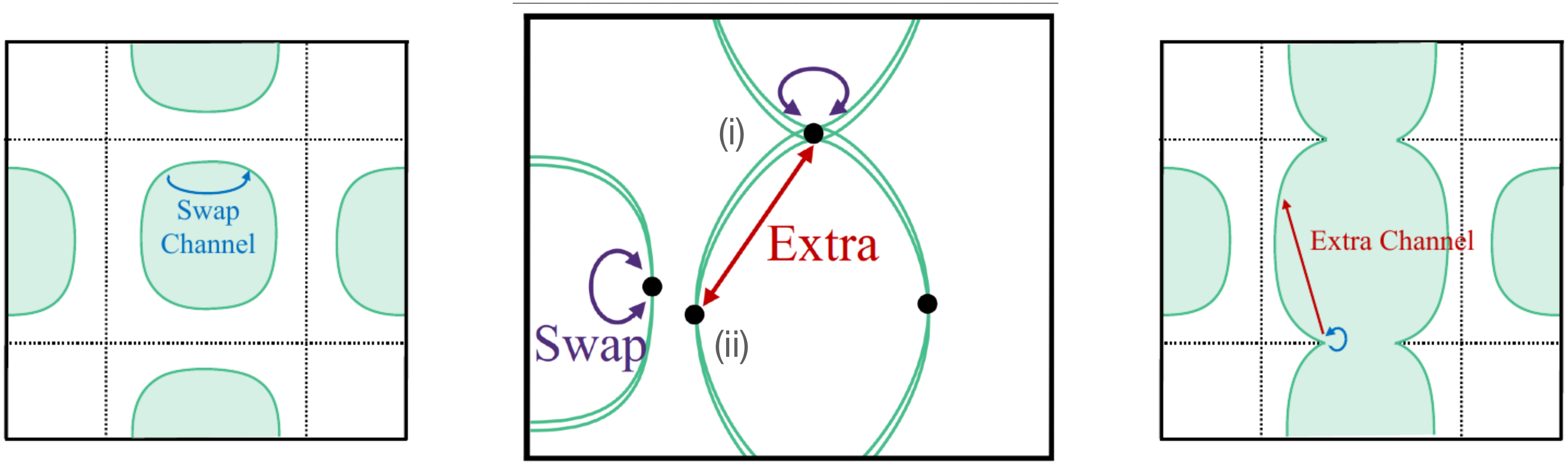


Optical conductivity of VHS: σ^J



- Swap/cooper channel: $\mathrm{Im}[\Xi^R_{SE} + \Xi^R_V + \Xi^R_{AL}] = 0.$
- Extra channel is non-zero: $\mathrm{Re}[\sigma^J] \sim \dfrac{\mathrm{Im}[\Xi^R_{SE} + \Xi^R_V]}{\Omega} \sim \dfrac{|J|^2}{\Omega}$





- Lifshitz transition at zone boundary, include higher-order dispersion:

$\epsilon_k = k_x^2 - ak_y^2 + bk_y^4(b > 0,\ a = 1).$

- Two geometrically inequivalent solutions shifting by small momentum $\mathbf{q}$:

(i) around VHS: $\epsilon_{\mathbf{k}} = k_x^2 - ak_y^2$ (no linear term!)

(ii) convex FS region: $\epsilon_{\mathbf{k}} \simeq -k_x/\sqrt{b} + ak_y^2$

- Swap Channel: $(\text{SE+V})_{(\text{i})}+\text{AL}_{(\text{i})}$

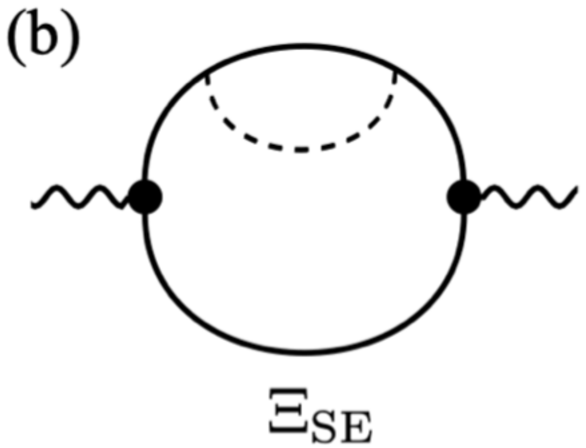
$$\mathrm{Re}[\sigma^J_{xx}(\Omega, T=0)] = \frac{\mathrm{Im}[\Xi(i\Omega_m,0)]\,|_{i\Omega_m\to\Omega+i0^+}}{\Omega} \simeq \frac{N|J|^2\Lambda_\theta}{4\pi^3\sqrt{a}\,|\Omega|} - \frac{N|J|^2\Lambda_\theta}{4\pi^3\sqrt{a}\,|\Omega|} = 0.$$

- Extra Channel: $(\text{SE+V})_{(\text{i})}+\text{AL}_{(\text{ii})}$ with $\text{AL}_{(\text{ii})}$=0

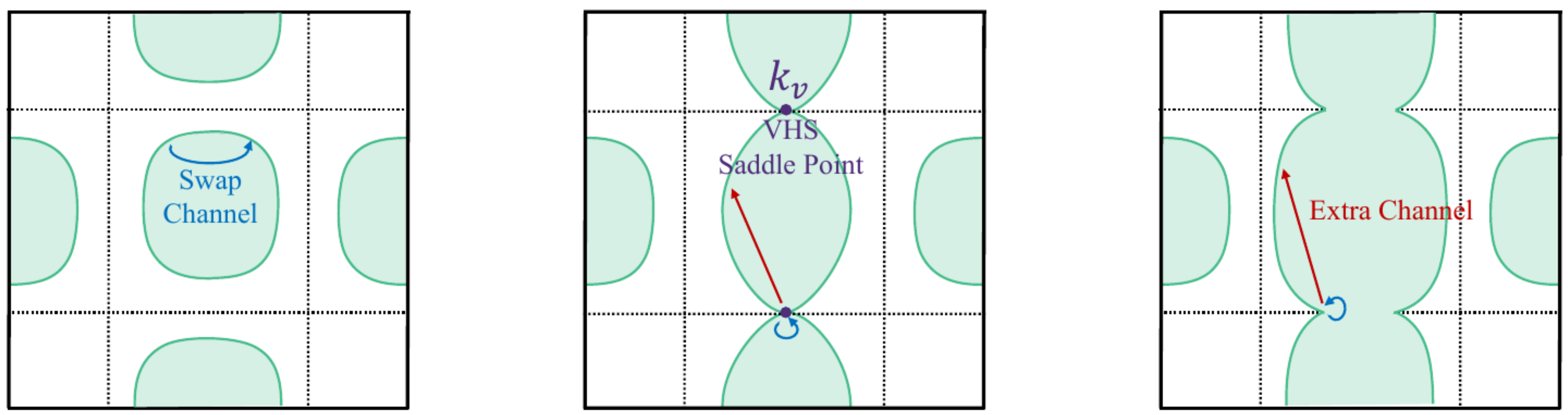
$$\mathrm{Re}[\sigma^J_{xx}(\Omega, T=0)] = \frac{\mathrm{Im}[\Xi(i\Omega_m,0)]\,|_{i\Omega_m\to\Omega+i0^+}}{\Omega} \simeq \frac{N|J|^2\Lambda_\theta}{4\pi^3\sqrt{a}\,|\Omega|}.$$

Optical conductivity of VHS: $\sigma^{J'}$

- Only Ξ_{SE} contributes ----> $\mathrm{Re}[\sigma^{J'}] \sim \dfrac{|J'|^2\Lambda^2}{\Omega}$;
- Similar to the FS case.



- Overall optical conductivity at $T = 0$: $\mathrm{Re}[\sigma(\Omega, T = 0) \sim \dfrac{|J|^2 + |J'|^2\Lambda^2}{\Omega}$



Optical conductivity of VHS at $T \neq 0$

- Let's consider the J'-dominant regime with marginal Fermi liquid:

$$\Pi(i\Omega_m, \boldsymbol{q}) - \Pi(0) \simeq -\frac{|J'|^2\Lambda^2}{2\pi^3}\frac{N}{N'}|\Omega_m|,$$

overdamped boson

$$\Sigma(i\omega_n, \boldsymbol{k}) - \Sigma(0) \simeq i\frac{|J'|^2\Lambda}{8\pi^3}\omega_n \ln|\omega_n|,$$

MFL self-energy

- Renormalized boson propagator $z = 2$: $D^{-1}(\mathbf{q}, \omega) = q^2 + c_d|\omega| + m_b^2(T)$

  with the boson thermal mass $m_b^2(T) \simeq T \sim |\omega| \sim q^2$

- The low-frequency boson fluctuations do not become any less singular when $z = 2$;
- The Ω/T scaling applies ----> $\mathrm{Re}[\sigma^{J'}(T \gg \Omega)] = \dfrac{|J'|^2\Lambda^2}{T}$

  ----> $\rho(T) \sim \dfrac{T}{J'^2}$ linear-in-T resistivity from VHS in MFL regime.

Optical conductivity of VHS at $T \neq 0$

- Then, let's consider the J-dominant NFL regime

$$\Pi(i\Omega_m, \boldsymbol{q}) - \Pi(0) \simeq -\frac{|J|^2}{4\pi}\frac{N}{N'}\frac{|\Omega_m|}{|q_x^2 - aq_y^2|},$$

$$\Sigma(i\omega_n, \boldsymbol{k}) - \Sigma(0) \simeq -i\frac{|J|}{4\sqrt{\pi}}\sqrt{\frac{N'}{N}}\mathrm{sgn}(\omega_n)|\omega_n|^{1/2}.$$

- Renormalized boson propagator $z_b = 4$: $D^{-1}(\mathbf{q}, \Omega) = q^2 + c\frac{|\Omega|}{|q_x^2 - aq_y^2|} + m_b^2(T)$

- We carefully evaluate the full T-dependence in optical conductivity:
  (i) distribution functions;
  (ii) boson thermal mass $m_b(T)$.

$$\mathrm{Re}[\sigma_{xx}^J(\Omega \ll T)] = \frac{1 - e^{-\Omega/T}}{\Omega}\mathrm{Im}[\Xi(i\Omega_m, 0)]|_{i\Omega_m \to \Omega + i0^+}$$

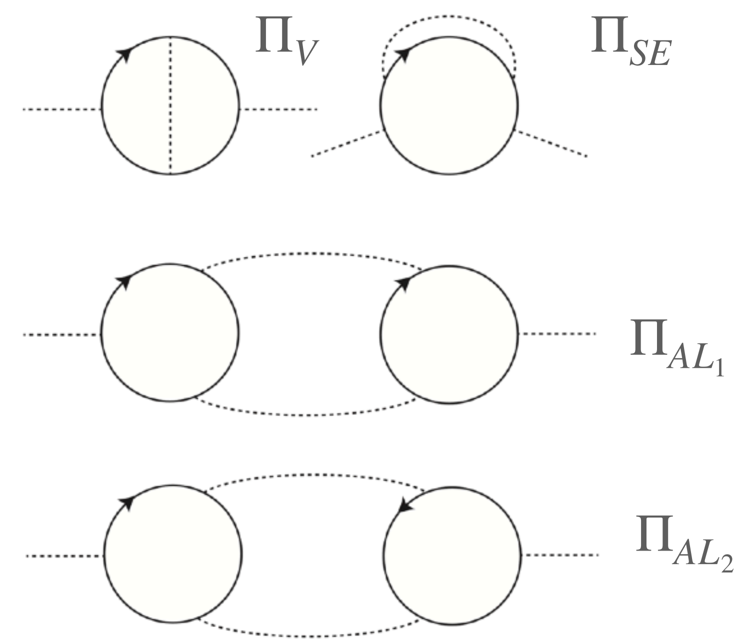
Effective thermal mass $m_b(T)$

- To calculate the conductivity at finite temperature, we need to consider the effects of the thermal mass $m^2(T) = m_b^2 - \Pi(0,0)$ which is severely renormalized by the higher order boson interactions.
- Self-consistent renormalization [by Moriya]:

  (1) contraction of external bosonic legs in boson interactions;

  (2) evaluate the boson mass using full boson propagators;

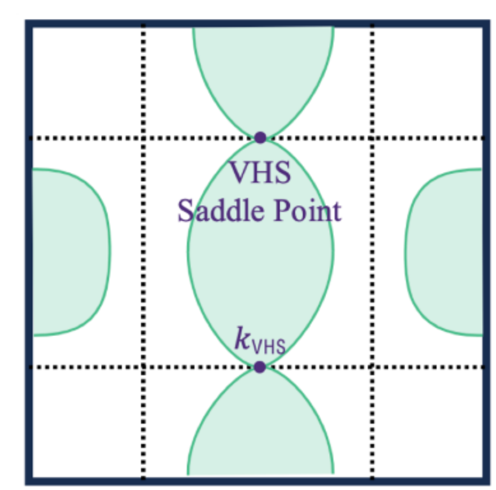
  (3) establish self-consistent equation for $m_b(T)$ and solve it.



Hartnoll et al., PRB (2014)

Near VHS, consider J interaction:

- One-loop diagram Π_0 gives constant;
- Higher-loop diagrams $\Pi_{SE} + \Pi_V + \Pi_{AL} = 0$;
- No T-dependence up to $o(|J|^4)$.



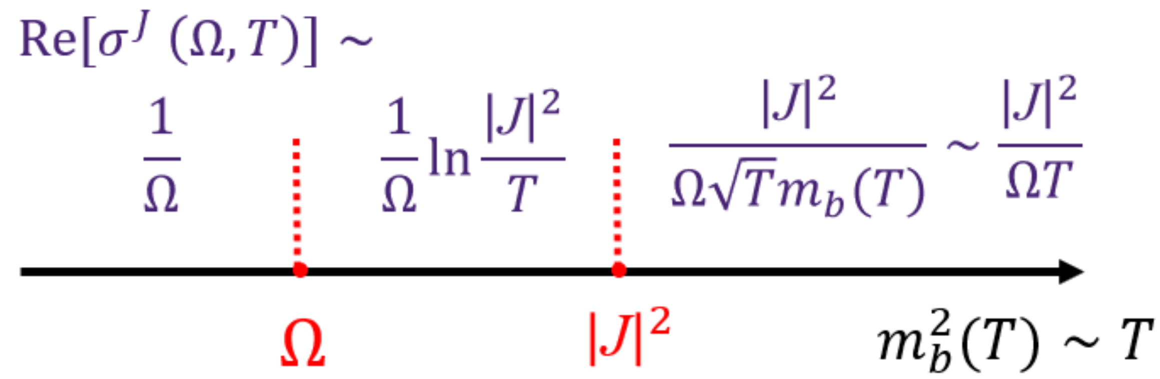
Rest of FS, consider J interaction:

- [Esterlis, Guo, Patel and Sachdev, PRB (2021)] ----> $m^2(T) \sim T\ln(1/T)$.

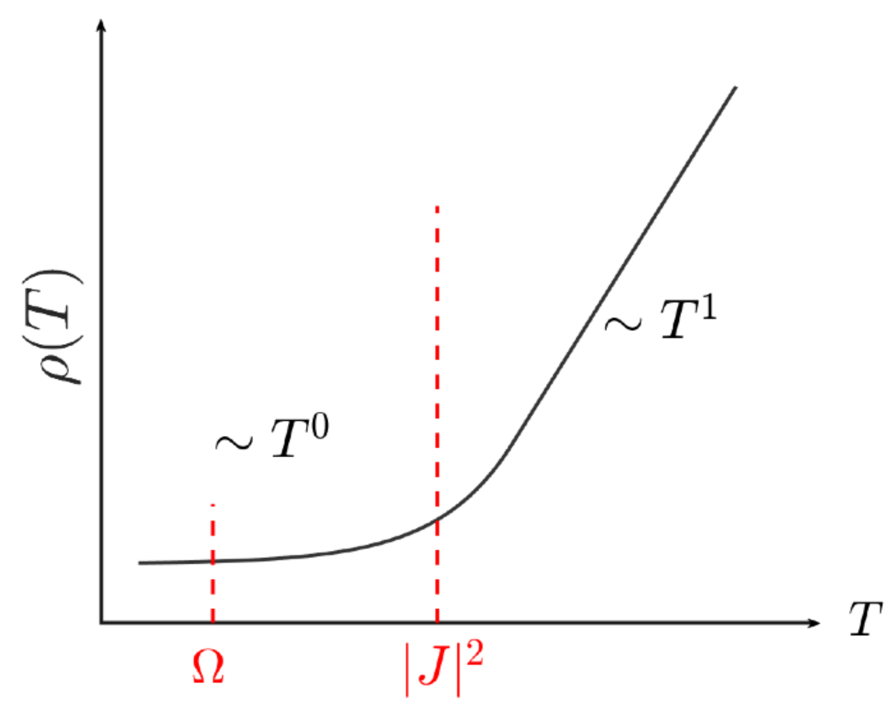
Overall, we have: $m_b^2(T) \sim T$ up to log correction.

Optical conductivity of VHS ($T \gg \Omega$)

- Swap channel =0; Extra channel ----> $\mathrm{Re}[\sigma^J(T \gg \Omega)]$:



- Schematic of $\rho(T)$ in distinct temperature regions:



$T \ll \Omega$ limit coincides with the $T = 0$ result

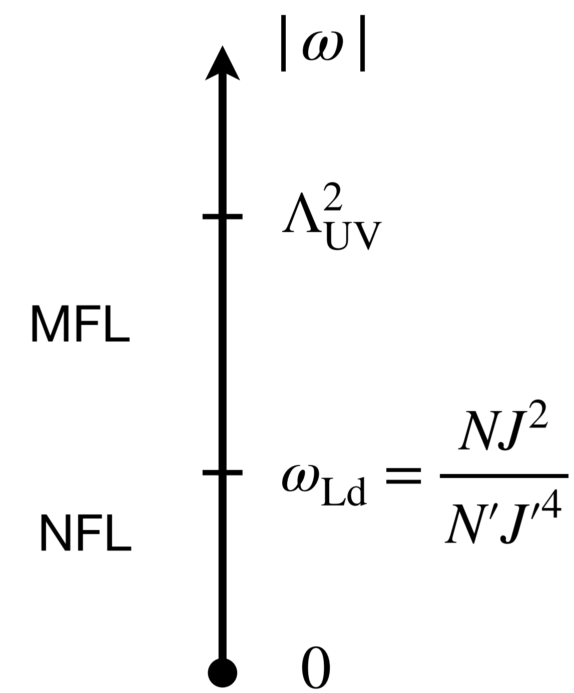
Persistent linear-in-T resistivity

- from VHS in J'-dominant MFL regime

$$\mathrm{Re}[\sigma^{J'}(T \gg \Omega)] = \frac{|J'|^2 \Lambda^2}{T}$$

- from VHS in J-dominant NFL regime

$$\mathrm{Re}[\sigma^{J}(T \gg \Omega)] = \frac{|J|^2}{\Omega T}$$



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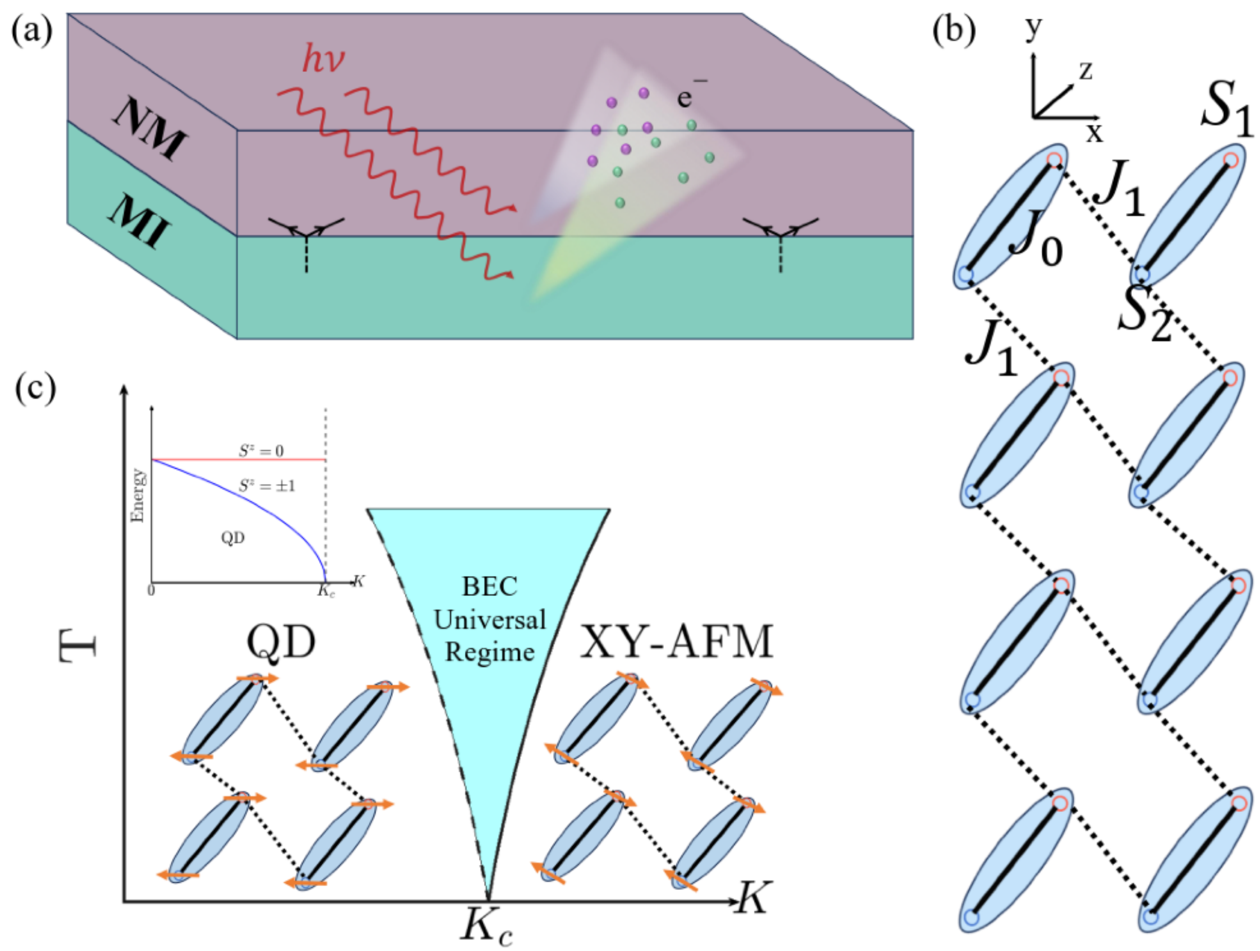
- Spintronics, interfacial DM interactions...

Summary and outlook

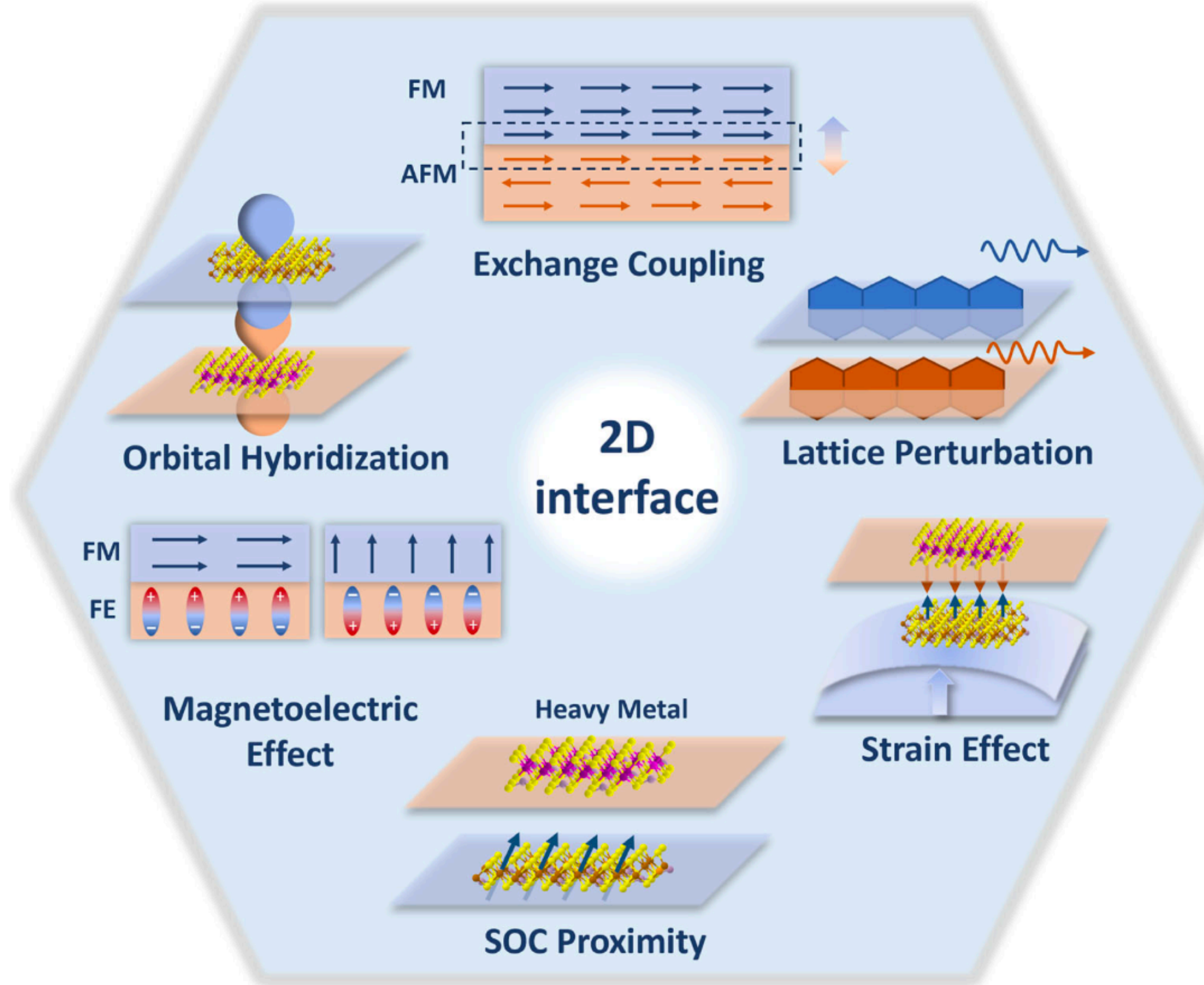
NFL/MFL in magnetic heterostructure



- Magnetic heterostructure interface: normal metal (NM) + magnetic insulator (MI);
- NM provides itinerant electrons: 2D Fermi surface with VHS points;
- MI provides critical boson: gapless triplon near BEC transition in dimerized AFM;
- QCP is reached when the triplon gap closes;
- Tuning the ratio J_0/J_1 via the uniaxial anisotropy (not external magnetic field).

Magnetic heterostructure

B. Zhang,..., Y. Y. Wu,
2D Magnetic heterostructures: spintronics and quantum future,
npj spintronics (2024)



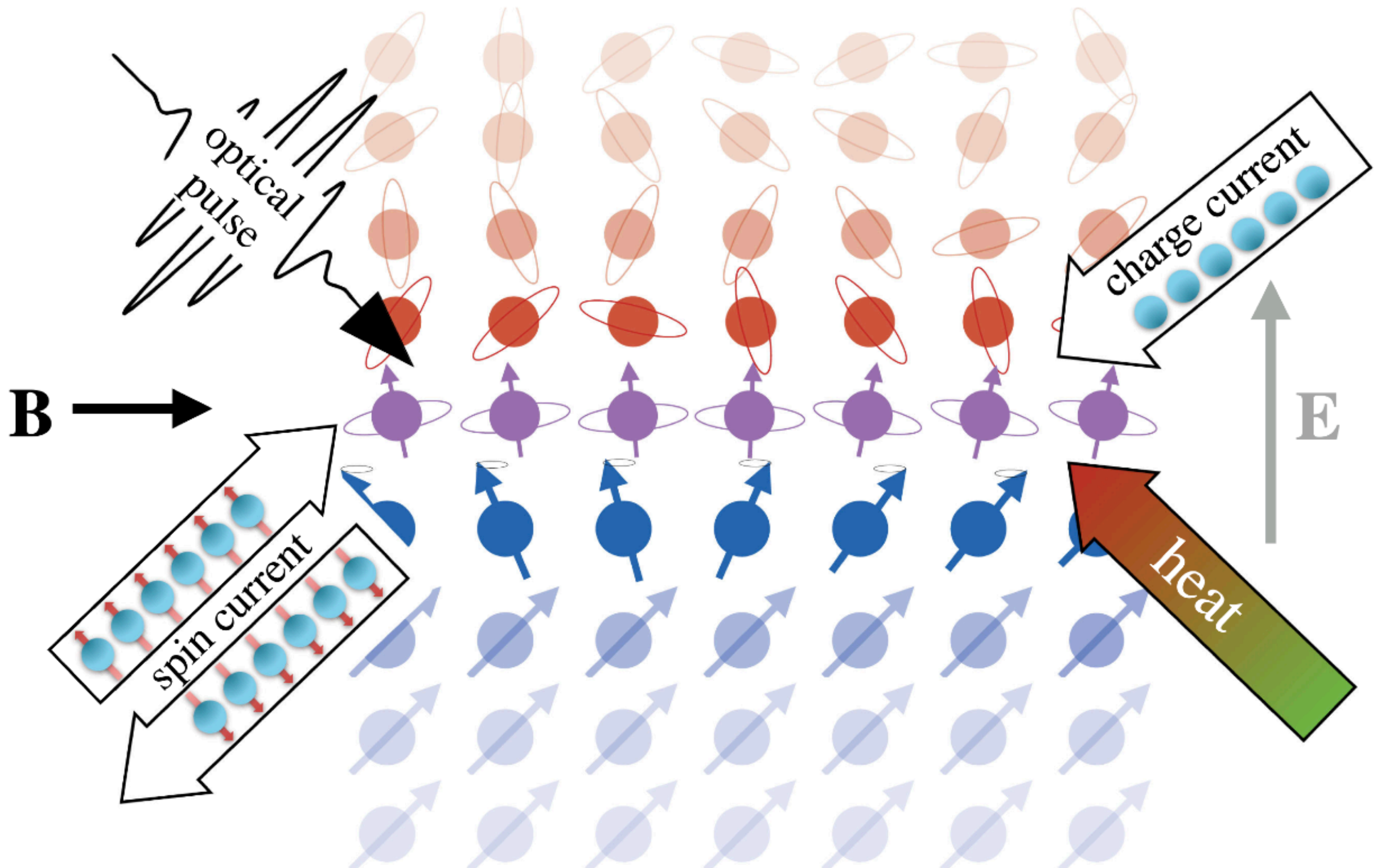
- The atomic-level 2D nature enables proximity effects;

- Proximity effects (orbital hybridization, exchange coupling...) ----> valuable, novel magnetic/electronic properties;

- Practical platform for co-integrating distinct physical ingredients by artificial design of heterostructures: e.g. magnetic memory and spintronic devices;

- Stacking and twisting of 2D materials expand the boundary of condensed matter physics: vdW heterostructure and “twistronics”.

Magnetic heterostructure and magnetic QCP

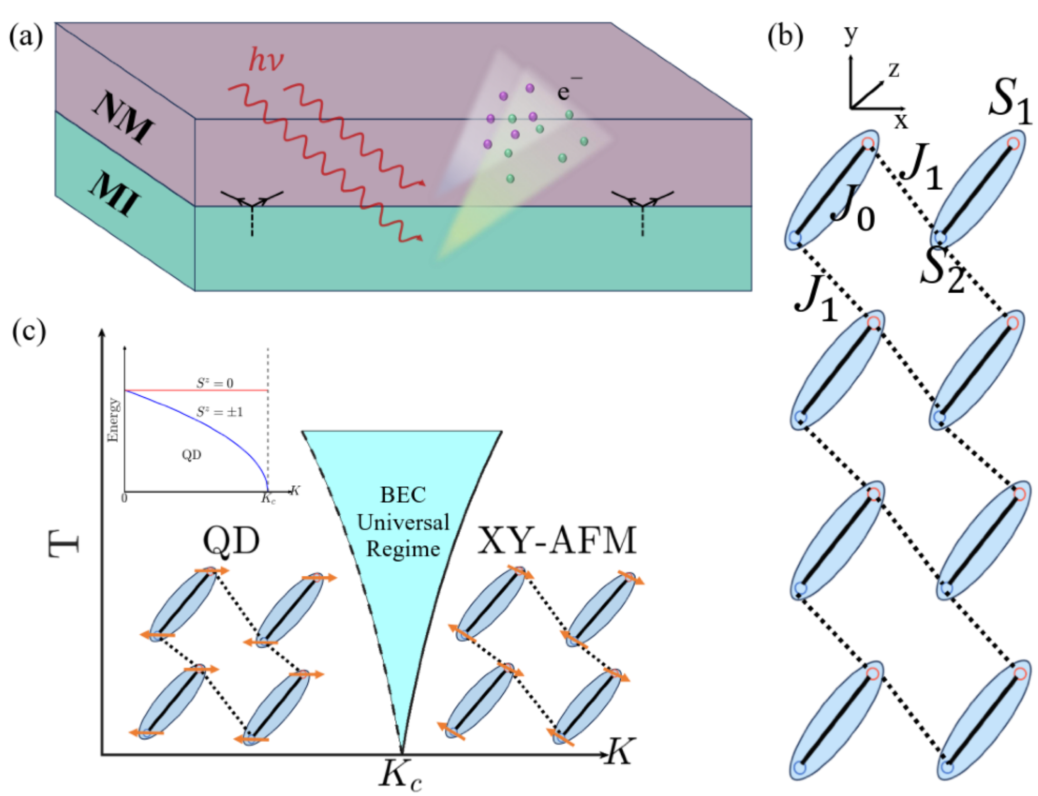


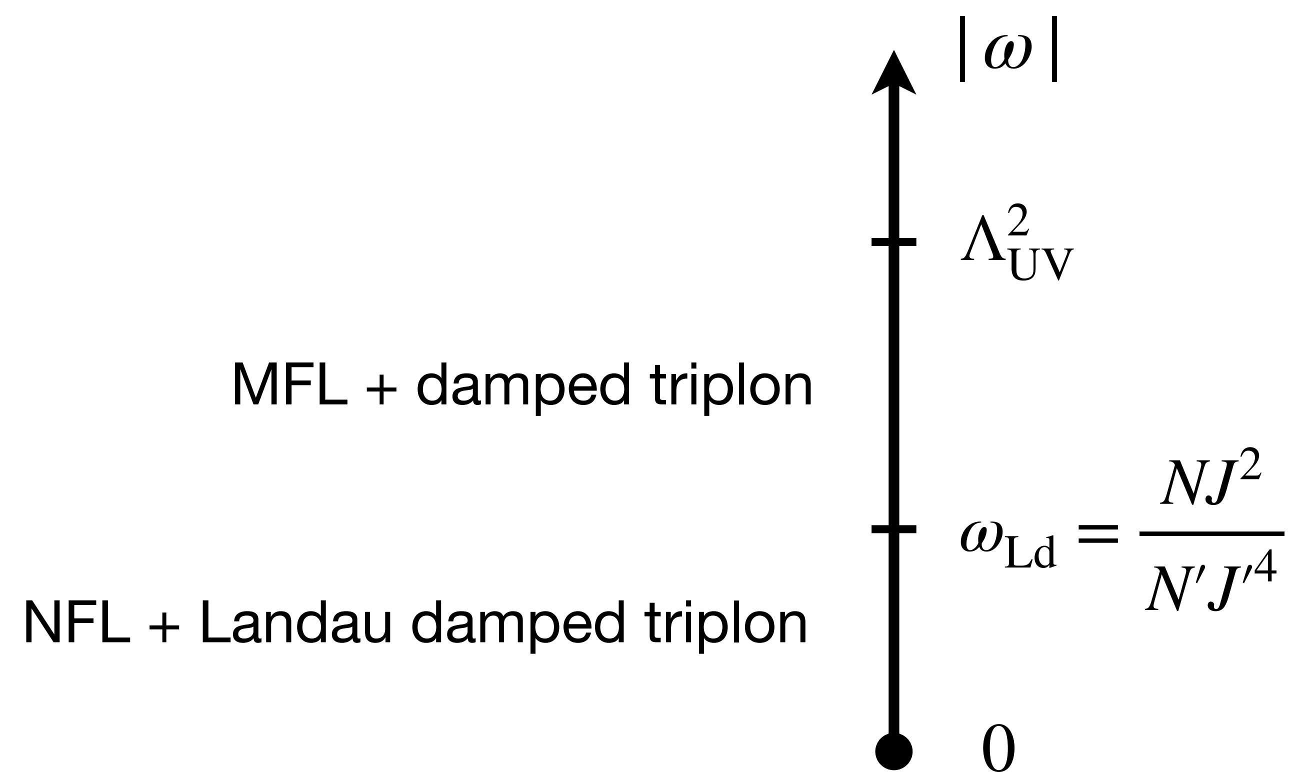
Hellman et al., ***Interface-induced phenomena in magnetism,*** Rev. Mod. Phys (2017)

- Lower dimensionality enables otherwise unattainable fabrication, manipulation and measurement as their 3D bulk counterparts;
- The strain, gating, light and magnetic field can couple with various internal degrees of freedom — charge, spin, orbit, lattice, etc. creating new magnetic properties, novel charge and spin transport;
- The versatility of magnetic heterostructure offers unprecedent tunability towards magnetic QCPs.

NFL/MFL in magnetic heterostructure

- VHS point dominates the NFL behaviors in low-energy limits;
- Large-N diagrammatic calculations ---->
  NFL in the low-energy limit: $|\omega| < \omega_{\mathrm{Ld}}$
  Marginal NFL at finite energies: $\omega_{\mathrm{Ld}} < |\omega| < \Lambda_{\mathrm{UV}}^2$;
- ARPES in magnetic heterostructure directly extracts the NFL nature in certain limits.





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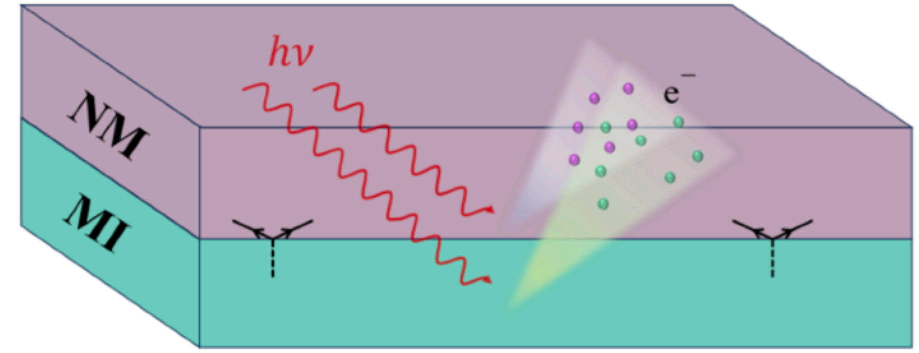
ARPES signal in magnetic heterostructure

V. Sunko et al., Sci. Adv. (2020)



- ARPES is a non-magnetic spectroscopic measurement for the electrons;
- Probe the spin correlations in magnetic heterostructure using ARPES:

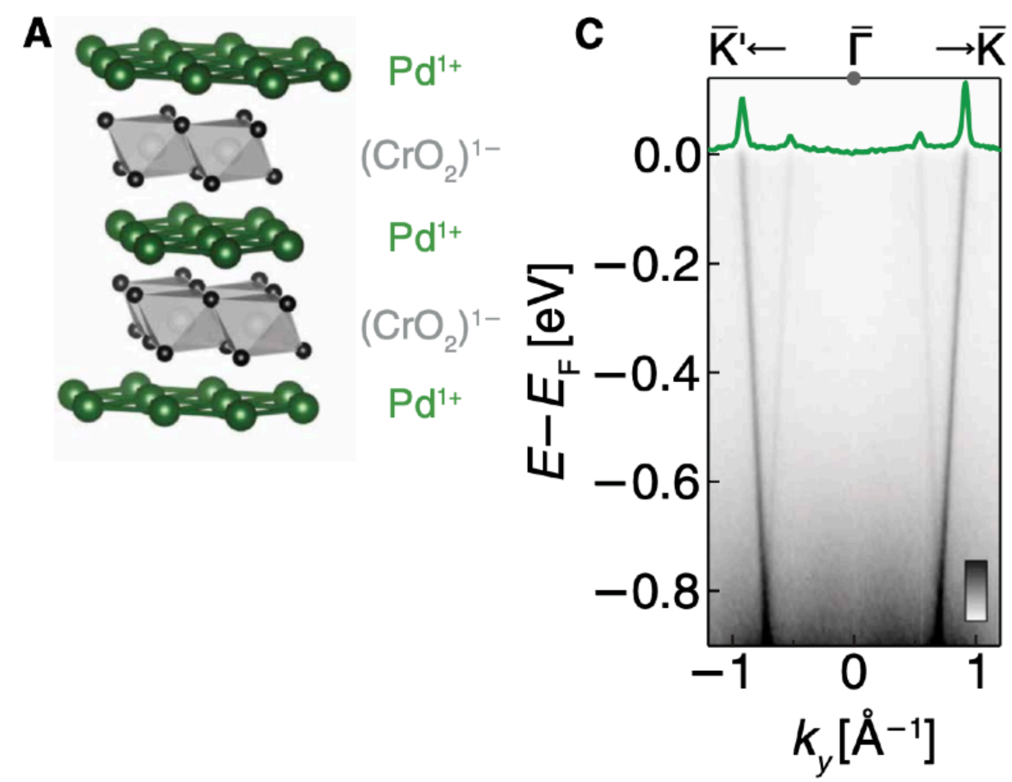
$$A(\omega, \boldsymbol{k}) = A_{\mathrm{NM}}(\omega, \boldsymbol{k}) + A_{\mathrm{MI}}(\omega, \boldsymbol{k}),$$

$$A_{\mathrm{MI}}(\omega, \boldsymbol{k}) \propto \int d\omega' d^2\boldsymbol{q}\ A_{\mathrm{NM}}(\omega', \boldsymbol{q}) S(\omega - \omega', \boldsymbol{k} - \boldsymbol{q}).$$

- For magnetic ordered MI, $S(\omega \to 0, Q)$ diverges:

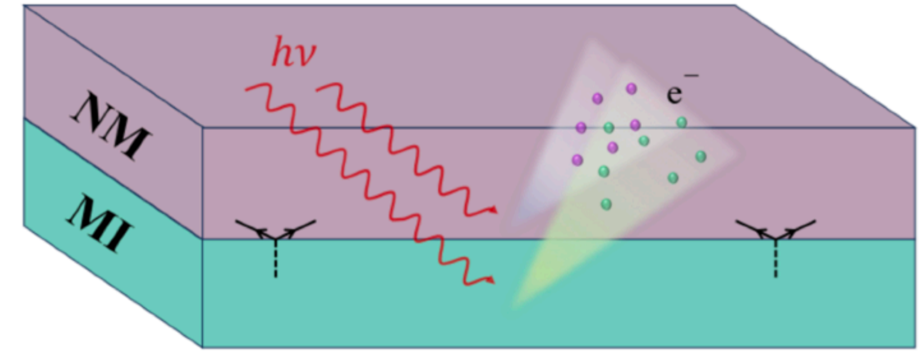
$$A_{\mathrm{MI}}(\omega, \boldsymbol{k}) \propto A_{\mathrm{NM}}(\boldsymbol{k} - \boldsymbol{Q}).$$

There's a spectral replica due to MI layer !



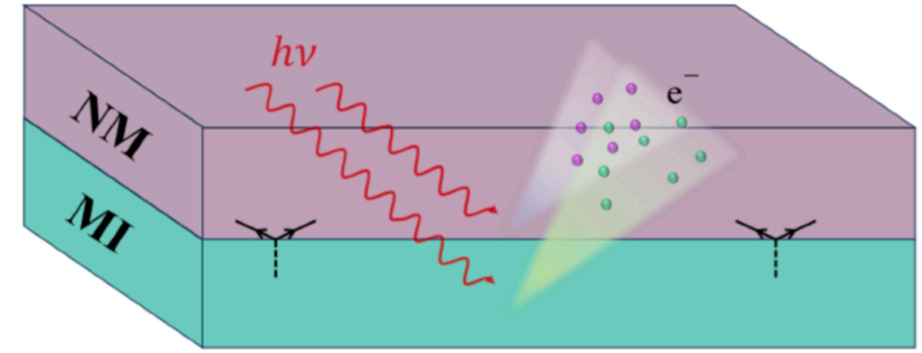
ARPES signal in magnetic heterostructure

C. Li, X. Yao and Gang Chen, PRL (2022)



- Probe the spin correlations in magnetic heterostructure using ARPES:
  $A(\omega, \boldsymbol{k}) = A_{\mathrm{NM}}(\omega, \boldsymbol{k}) + A_{\mathrm{MI}}(\omega, \boldsymbol{k})$,
  $A_{\mathrm{MI}}(\omega, \boldsymbol{k}) \propto \int d\omega' d^2\boldsymbol{q}\ A_{\mathrm{NM}}(\omega', \boldsymbol{q}) S(\omega - \omega', \boldsymbol{k} - \boldsymbol{q})$.
- For a quantum spin liquid (QSL), the spin correlation function S of MI exhibits a continuum in the momentum and frequency space;
- ***Crude assumption:*** NM is not changed by coupling to QSL and remains a Fermi liquid: $A_{\mathrm{NM}} \propto \delta(\epsilon_{\boldsymbol{k}} - \omega)$;
- The benign Fermi liquid ***extracts*** the spectral continuum of QSL in MI $A_{\mathrm{MI}}(\omega, \boldsymbol{k}) \propto S(\omega + \delta\omega, \boldsymbol{k})\delta\omega$.

ARPES signal for NFL



- Total spectral function of ARPES:

$$A(\omega, \boldsymbol{k}) = A_{\mathrm{NM}}(\omega, \boldsymbol{k}) + A_{\mathrm{MI}}(\omega, \boldsymbol{k})$$

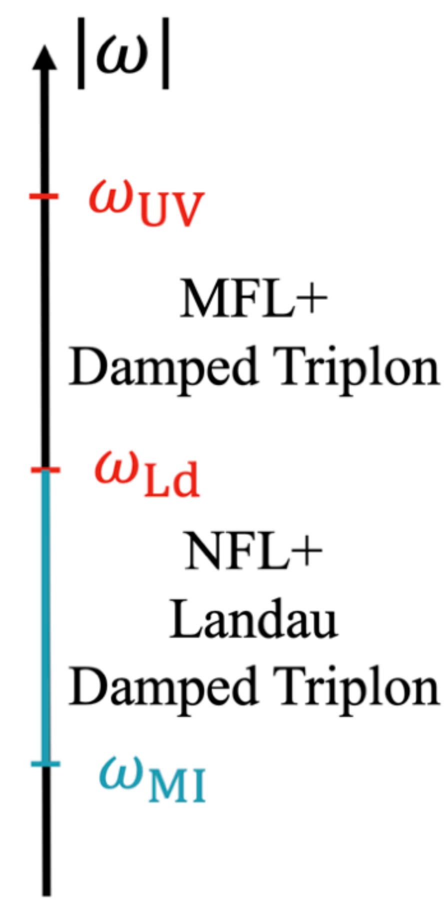
- Convolution of the NM Green function and MI spin correlation:

$$G_{\mathrm{MI}}(i\omega_n, \boldsymbol{k}) \propto \int d\omega_m d^2\boldsymbol{q}\ G_{\mathrm{NM}}(i\omega_m, \boldsymbol{q}) S(i\omega_n - i\omega_m, \boldsymbol{k} - \boldsymbol{q}) \propto \Sigma(i\omega_n, \boldsymbol{k})$$

- ARPES for magnetic heterostructure at QCP:

$$A_{\mathrm{MI}}(\omega, \boldsymbol{k}) \propto -\frac{\mathrm{Im}\Sigma(\omega, \boldsymbol{k})}{\pi} \sim |J| \sqrt{\frac{N'}{N}} |\omega|^{1/2}$$

- $A_{\mathrm{MI}}(\omega, \boldsymbol{k}) >> A_{\mathrm{NM}}(\omega, \boldsymbol{k})$:
  dominant in the boson large-N' with $N'/N \gg 1$!!



Summary

- We show distinct types of NFL/MFL can emerge near VHS at the Lifshitz transition;

- Non-vanishing optical conductivity is possible with spatially uniform Yukawa interaction J along owing to the emergence of VHS;

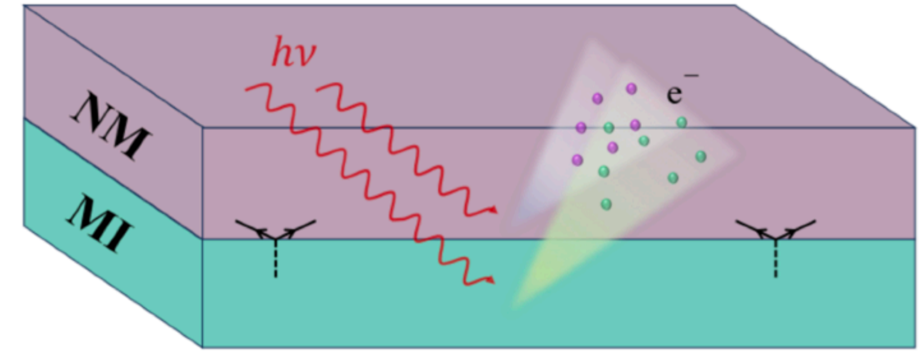
- Persistent linear-in-T resistivity in both NFL and MFL regimes.

- Magnetic heterostructure is a fertile land to study quantum critical phenomenon, mostly prominently, the NFL behaviors; Bilayer heterostructure provides physical ingredients for forming NFL at the interface.

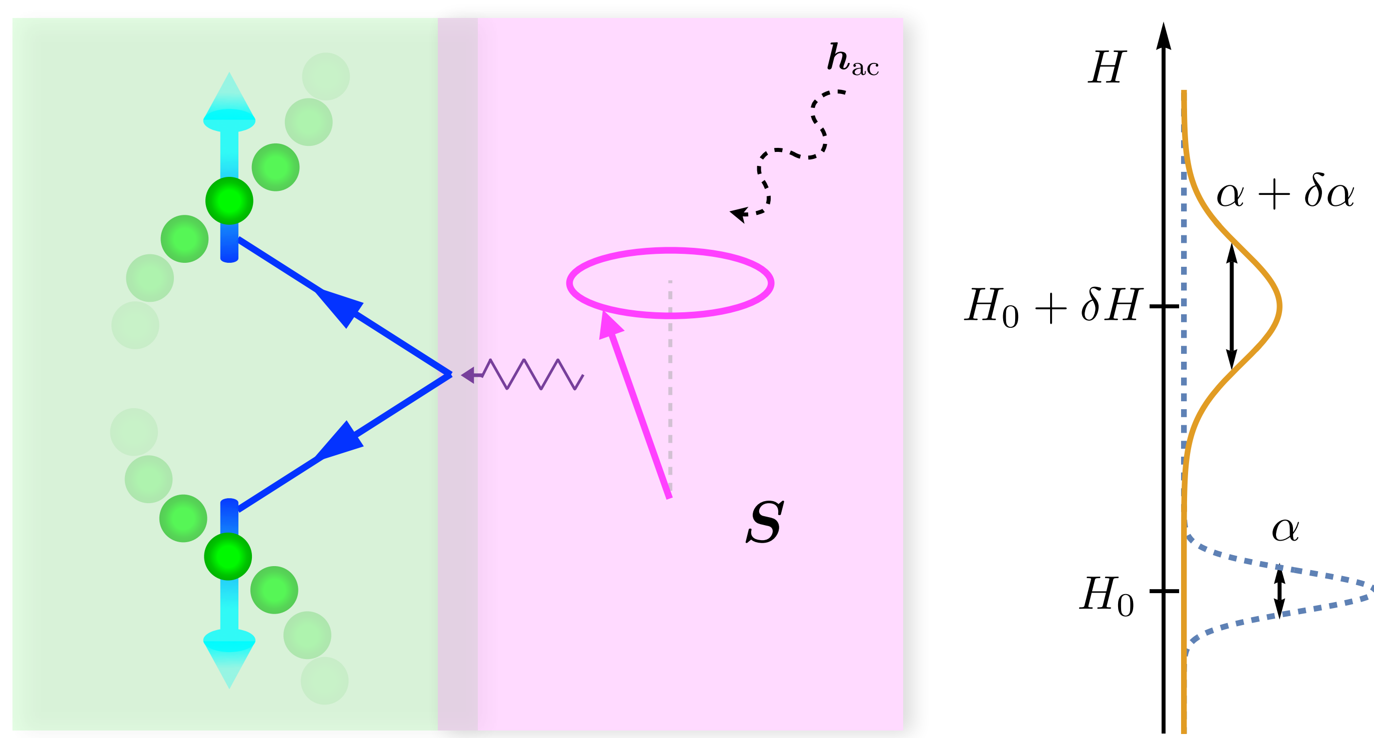
- Experimental probes for NFLs at the interface: optical conductivity, ARPES and spintronics methods ...

Outlook

interfacial effects

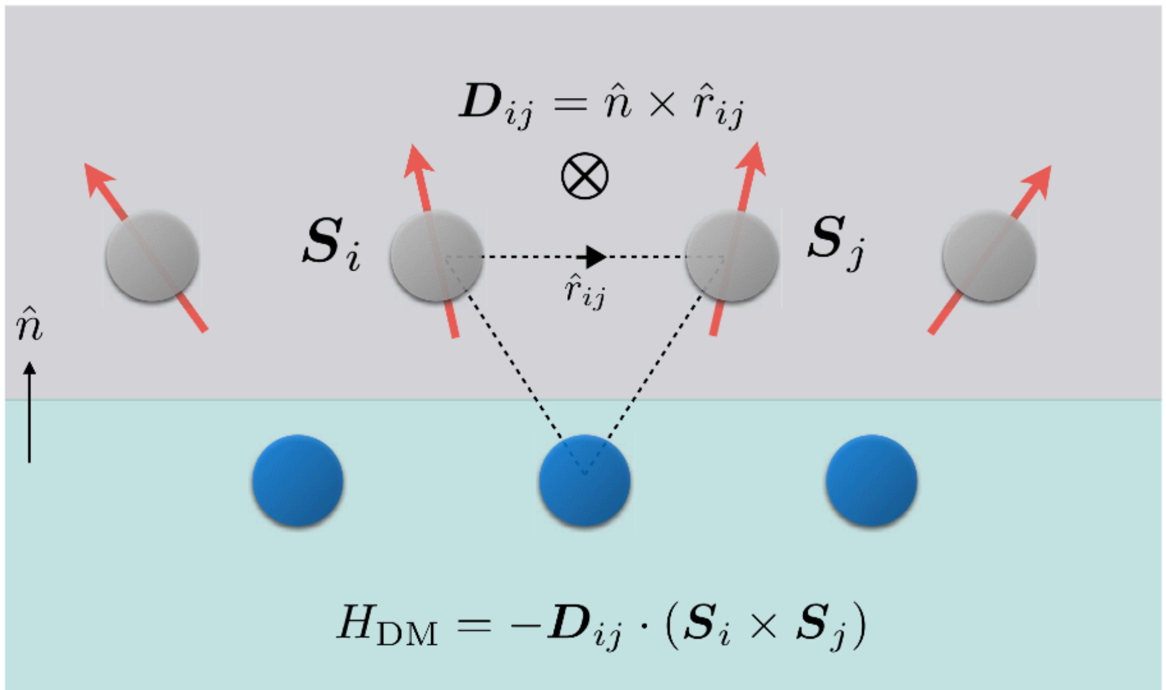


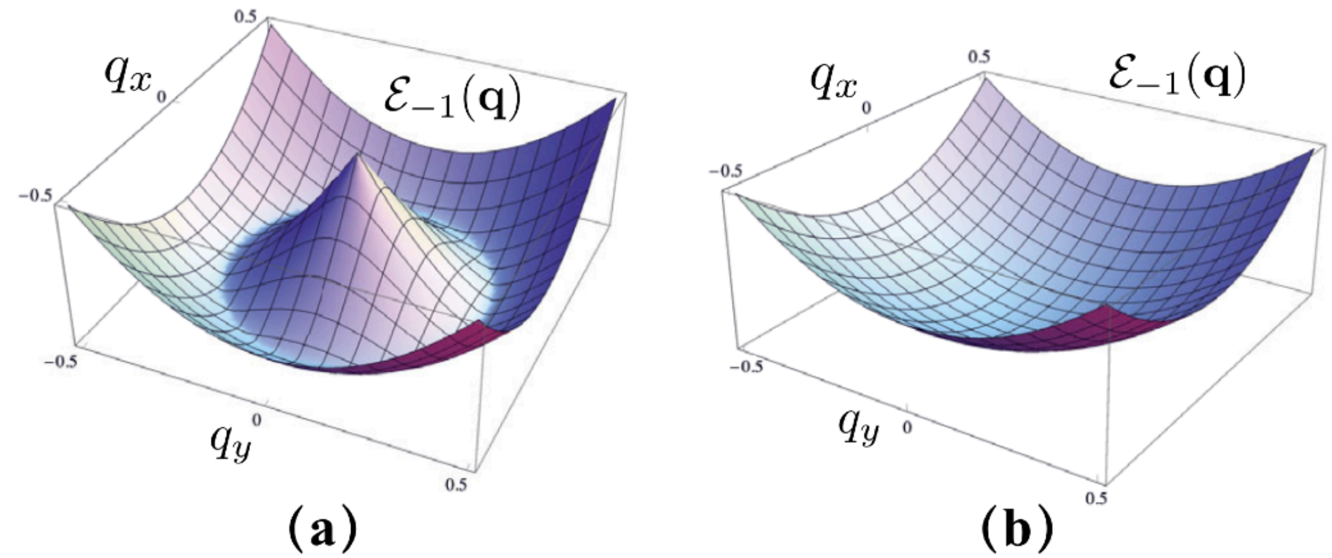
- Spintronics phenomenon at the magnetic heterostructure interface.
- Role of DM interactions at the magnetic heterostructure interface;



XTZ, ... and Mamoru Matsuo, arXiv: 2308.02189

Infinite critical boson NFL in magnetic heterostructure





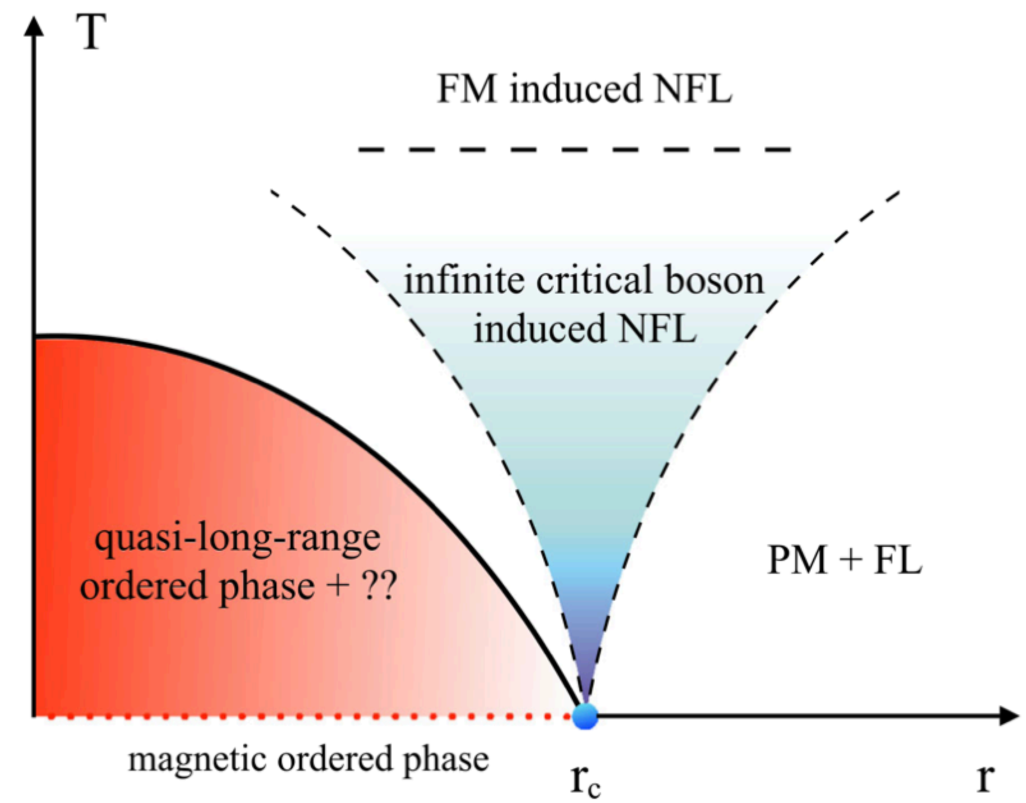
critical boson ring with DMI

$$\mathcal{L}_{\rm B}[\vec{\phi}] = \int d^2\boldsymbol{x} \frac{1}{2}\Big\{\vec{\phi}\cdot(r - J\nabla^2)\vec{\phi} + \frac{u}{2}(\vec{\phi}^2)^2 + D[\hat{y}\cdot(\vec{\phi}\times\partial_x\vec{\phi}) - \hat{x}\cdot(\vec{\phi}\times\partial_y\vec{\phi})]\Big\},$$

- Heterostructure interface breaks inversion symmetry
  ----> Dzyaloshinskii-Moriya interaction (DMI);
- DMI is odd under inversion leading to a continuously degenerated boson minima (for the lowest energy branch): ***critical boson ring***;
- Approaching QCP, infinite many bosons become critical and the wild quantum fluctuation induces novel type of NFL behavior in quantum critical regime.

XTZ and Gang Chen, 2102.09272
XTZ and Gang Chen, 2109.06594
Zhiming Pan and ***XTZ***†, 2205.03818

Infinite critical boson NFL in magnetic heterostructure



- We adopt the self-consistent renormalization (SCR) to deal with the fluctuation of infinite critical bosons;
- The bare boson mass acquires a peculiar temperature dependence: $m_b^2(T) = T^\alpha$, $\alpha = 5/9$;
- This exponent is different from conventional 2D QCPs: FM ($\alpha = 4/3$) and AFM ($\alpha = 3/2$) cases. It crossovers to the FM QCP at finite $T \sim \mathcal{O}(D^2/J)$;
- NFL behavior induced by the critical boson ring: $\rho(T) \sim -\mathrm{Im}\Sigma(\boldsymbol{k}_F, \omega = 0; T) \sim T^2/m_b^2(T) \sim T^{2-\alpha} \sim T^{1.44}$

ELSEVIER

Physics Reports
Volume 1070, 12 June 2024, Pages 1-59

Thermal Hall effects in quantum magnets

Xiao-Tian Zhang [b c 1], Yong Hao Gao [c 1], Gang Chen [a c d]

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https://doi.org/10.1016/j.physrep.2024.03.004

60 pages review article
on ***Physics Reports***

Abstract

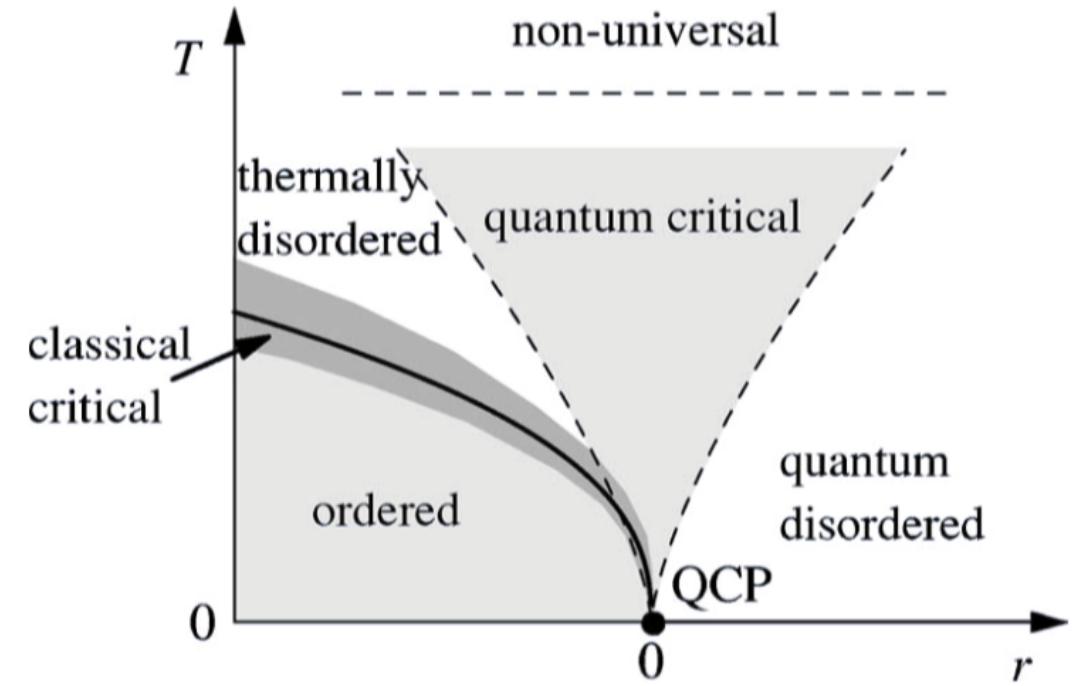
In the recent years, the thermal Hall transport has risen as an important diagnosis of the physical properties of the elementary excitations in various quantum materials, especially among the Mott insulating systems where the electronic transports are often featureless. Here we review the recent development of thermal Hall effects in quantum magnets where all the relevant excitations are charge-neutral. In addition to summarizing the existing experiments, we pay a special attention to the underlying mechanisms of the thermal Hall effects in various magnetic systems, and clarify the connection between the microscopic physical variables and the emergent degrees of freedom in different quantum phases. The external magnetic field is shown to modify the intrinsic Berry curvature properties of various emergent and/or exotic quasiparticle excitations in distinct fashions for different quantum systems and quantum phases, contributing to the thermal Hall transports. These include, for example, the conventional ones like the magnons in ordered magnets, the triplons in dimerized magnets, the exotic and fractionalized quasiparticles such as the spinons and the magnetic monopoles in quantum spin liquids. We review their contribution and discuss their presence in the thermal Hall conductivity in different physical contexts. We expect this review to provide a useful guidance for the physical mechanism of the thermal Hall transports in quantum magnets.

Thanks for your attention

and your participation in the symposium!

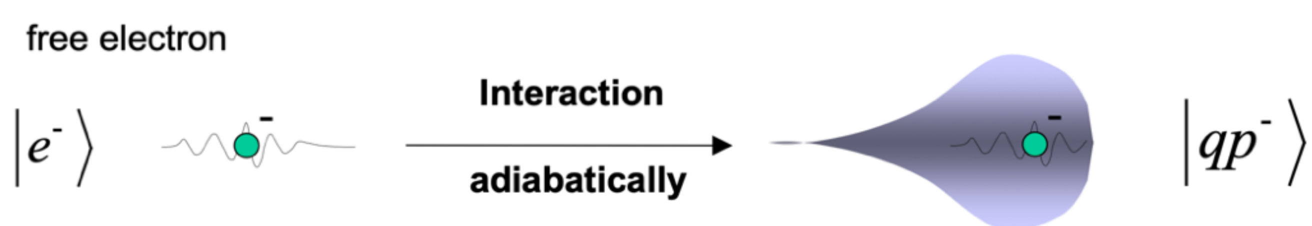
Supplementary Materials

Magnetic QCP and non-Fermi liquids



- We focus on the strong correlation effects near the magnetic QCP in the heterostructure (less studied than the magnetic orders);

- The wild quantum fluctuation near the QCPs induces long-range interactions for the itinerant electrons;



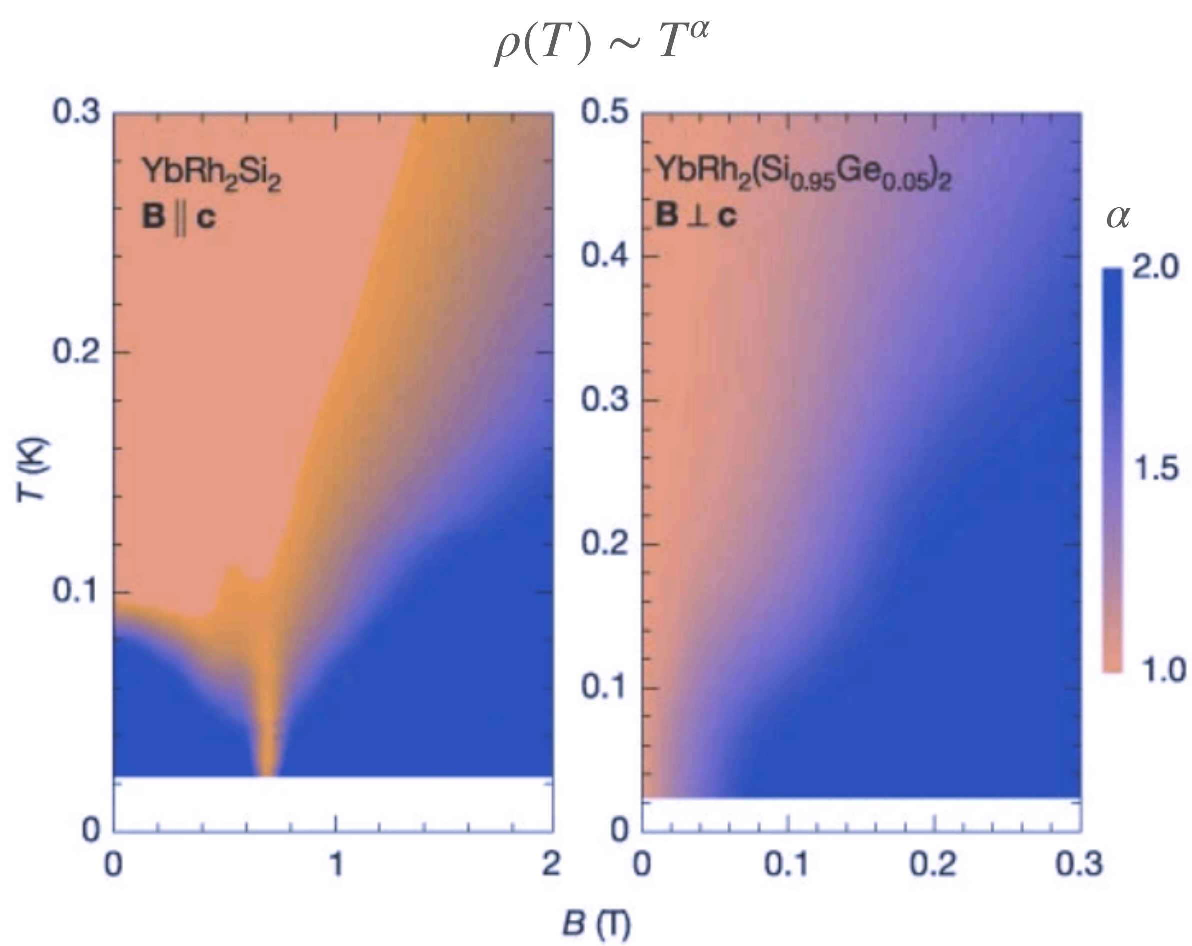
- Most dramatic manifestation of many-body correlation: breakdown of coherent quasiparticles ----> NFL;

- Interface provides necessary ingredients for NFLs: gapless fermions + critical bosons;

- ***NFL at the interface calls for design of novel experimental schemes for detection !!***

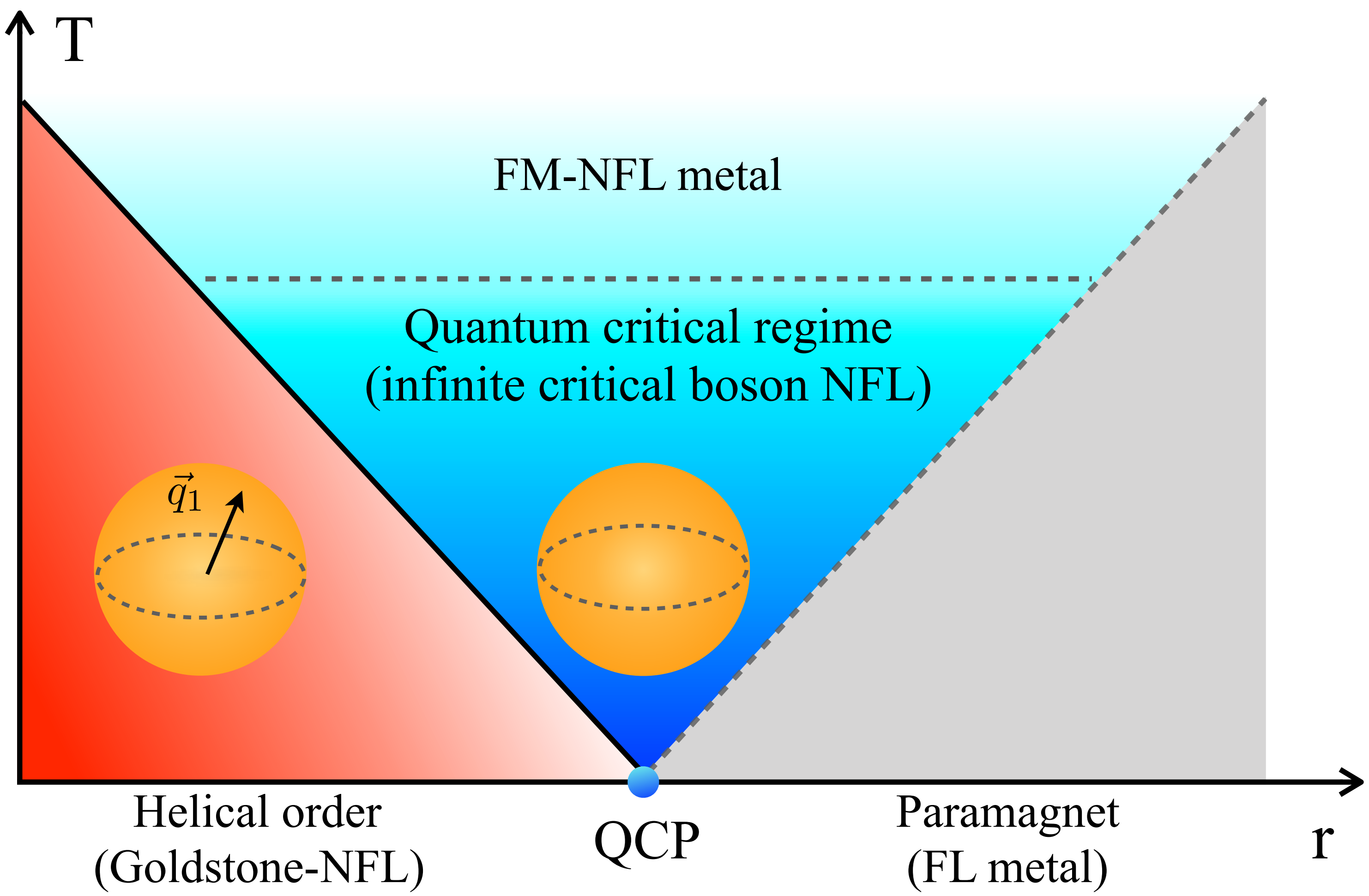
Non-Fermi liquid **breakdown of Landau's paradigm**

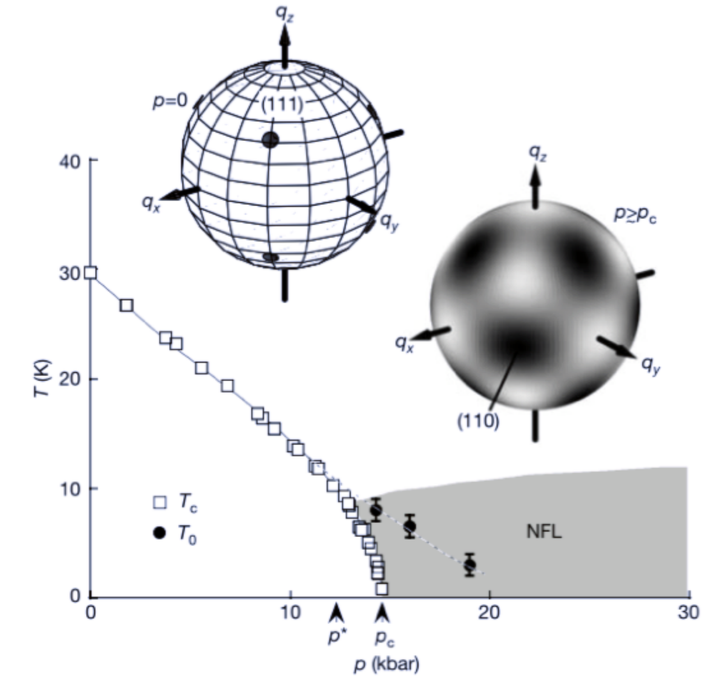
- Electron Green function: $G(\mathbf{k},\omega) = \left[\omega - \epsilon_{\mathbf{k}} - \Sigma(\mathbf{k},\omega)\right]^{-1}$;
- Quasiparticle decay rate: $\mathrm{Im}\Sigma(\mathbf{k}_F,\omega) \sim \omega^2$ for $\omega \to 0$;
- Quasiparticle weight: $Z_{\mathbf{k}_F}(\omega) = \left[1 - \partial_\omega \mathrm{Re}\Sigma(\mathbf{k}_F,\omega)\right]^{-1}_{\omega\to 0} \neq 0$;
- Fermi liquids: $\rho(T) \sim T^2$, $c_V(T) \sim T$, $\chi(T) \sim \mathrm{const}\ \ldots$
- NFL definition: $\omega \to 0$, $\mathrm{Im}\Sigma(\mathbf{k}_F,\omega) \sim \omega^a$, $0 < a \leq 1$ and $Z_{\mathbf{k}_F}(\omega) \to 0$;
- Universal power-low behavior of physical quantities at low-T: $\rho(T) \sim T^\alpha$, $1 \leq \alpha < 2$.
- High-T_c superconductors, itinerant magnets, heavy fermions, twisted bilayer graphene...



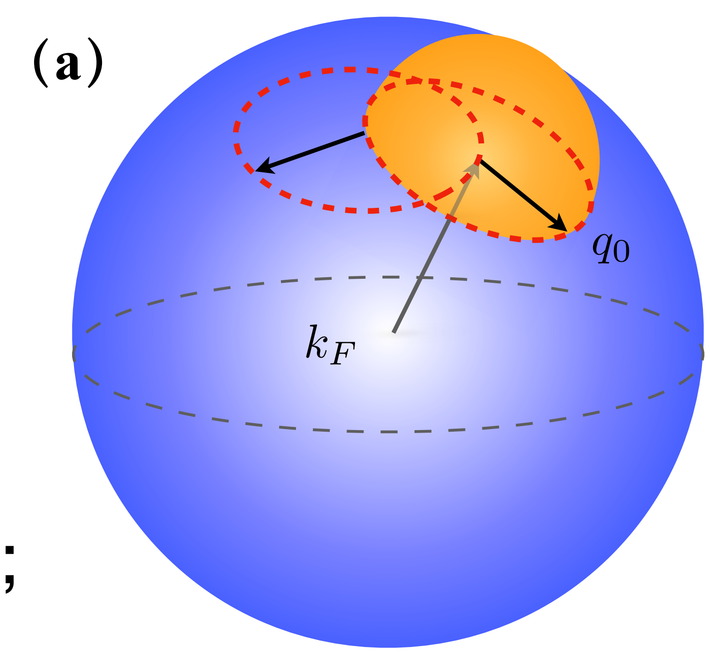
Novel type of NFLs

critical boson surface ($Q \neq 0$)





Pfleiderer et al, Science (2001&2004)



- Hertz-Millis theory and self-consistent renormalization;
- Electronic resistivity $\rho(T) \sim T^{\alpha}$, $\alpha = 1.4$.

XTZ and Gang Chen, npj Quantum Mater. (2023)
XTZ and Gang Chen, Quantum Front (2023)
Zhiming Pan and ***XTZ***†, Nucl. Phys. B (2024)